EXPANDED VERSION

In this section, we consider two (or more) dependent variables that depend on a third variable (the independent variable). Usually, the independent variable will be time.

Review: Relating Rates of Change Unless otherwise indicated, assume each variable represents an implicit function of time t.

1. If $p^2 + 3p + 1 = 12py$, find an equation relating $\frac{dp}{dt}$ with $\frac{dy}{dt}$.

2. If $\sqrt{x^2 + 3} = u$, find an equation relating $\frac{du}{dt}$ with $\frac{dx}{dt}$.

Summary of the Related Rates Process (V^2P GED S^3)

1. (V^2) Identify all of the *variables* and all of the *values* given in the problem. Make note of which values are for a "specific time" and which are "all the time" (i.e. are constants).
2. (P) Draw a *picture* if one is appropriate to the scenario; consider drawing a "starting time" picture and a "later on" picture. Label your diagram with the appropriate variables.
3. (G) Goal. In terms of the variables, what are you trying to find?
4. (E) Find an *equation* relating the dependent variables. (Is the equation given in the problem?) You may use your *constants*, but you may not use your *specific time* values. Sometimes you will need more than one equation.
5. (D) *Differentiate* both sides of the equation with respect to time. (Remember to follow differentiation rules.)
6. (S^3) *Substitute* known quantities and rates into the equation, and *solve* for the unknown rate. Write a *sentence* which interprets your solution in context, including units.

1. Imagine that you are blowing up a balloon, and its shape is perfectly spherical. Suppose that you are blowing air into this spherical balloon at a rate of 2π cubic centimeters per minute. After 3 minutes, at what rate is the radius of the balloon increasing?

2. A police cruiser sits beside the road on a straight portion of Macon Highway, and the cruiser is positioned 6 feet from the road. When you are 100 feet down the road from the officer, the officer points his radar at you and determines that the distance between him and your car is decreasing at 63 miles per hour. The speed limit is 55 miles per hour. How fast are you driving at that instant? Are you speeding? (Hint: check your units.)

3. Soft-serve frozen yogurt is being dispensed (straight down - no swirling!) into a waffle cone (in the shape of a right circular cone) at a rate of 10 cubic centimeters per second. If the waffle cone has height 15 centimeters and radius 2.5 centimeters at the top, how quickly is the yogurt in the cone rising when the height of the yogurt is 5 centimeters?

Advice For Related Rates

Things to Think About

- Make sure that your units are consistent. (If your units are not consistent, convert them.) Include units in your answer whenever appropriate.
- Pay attention to whether each quantity is increasing (positive rate of change) or decreasing (negative rate of change).
- The distance formula and the Pythagorean theorem are the same thing, said in slightly different ways.
- The perimeter of a shape is the length of a piece of string that wraps all the way around the shape, exactly one time.

Formulas to Remember

- Distance between (x_1, y_1) and (x_2, y_2): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Triangles
  - area: $A = \frac{1}{2}bh$
  - law of cosines: $c^2 = a^2 + b^2 - 2ab\cos(\theta)$
  - law of sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$
  - Be able to use properties of similar triangles.
  - right triangles and trig: SOH-CAH-TOA
  - Make sure you know the values of all of the trig functions at key angles (refer to unit circle): $0, \dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{3}, \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \ldots$
  - Your trig differentiation formulas assume that your angle is in radians. (Why?)
- Circles
  - area: $A = \pi r^2$
  - circumference: $C = 2\pi r$
  - Equation of the circle of radius r centered at (h, k): $(x - h)^2 + (y - k)^2 = r^2$
- Rectangles
  - area: $A = lw$
  - perimeter: $P = 2l + 2w$
- Cone volume: $V = \frac{1}{3}\pi r^2 h$
- Cylinder volume: $V = \pi r^2 h$
- Spheres
  - volume: $V = \frac{4}{3}\pi r^3$
  - surface area: $S = 4\pi r^2$
- Rectangular prisms
  - volume: $V = lwh$
  - surface area: $S = 2lw + 2wh + 2lh$

1. The Doctor decides to take Dr. Royal to space. They step into the TARDIS (time/space traveling machine), which is sitting on the Myers quad 100 feet from Dr. Royal's cat Walter, who is taking a nap, and the TARDIS begins traveling straight up at a rate of 30 ft/sec. At what rate is the angle of elevation, measured from Walter, changing when the TARDIS has been in flight for 5 seconds?

2. Sayonita is bored and decides to pour an entire container of salt into a pile on the kitchen floor. She pours 3 cubic inches of salt per second into a conical pile whose height is always two-thirds of its radius. How fast is the height of the conical salt pile changing when the radius of the pile is 2 inches?

3. A softball diamond is a square with each side measuring 60 feet. Suppose a player is running from second base to third base at a rate of 3.5 feet per second. At what rate is the distance between the runner and home plate changing when the runner is halfway to third base? How far is the runner from home plate at this time?

4. A particle moves along the curve $y = 7\sqrt{2x-1}$. When the particle is at the point $(3, 7\sqrt{5})$, the distance from the particle to the origin is decreasing at a rate of 2 units per second. What is the rate of change of the y-coordinate of the particle at that instant?

5. Suppose that Stuart is 6 feet tall and is walking towards a 20-foot streetlight. How fast is the length of Stuart's shadow changing when he is 17 feet from the streetlight and is approaching the light at a speed of 4 feet per second?

6. In the piston assembly in the figure below, a 7-inch connecting rod is fastened to a crankshaft of radius 3 inches. The crankshaft rotates counterclockwise at a constant rate of 200 revolutions per minute. Find the velocity of the piston when $x = \pi/3$.