Overview

Below is a worksheet with several worked examples of u-subs integrals

Please complete and turn in full-written solutions to:

- Q1a-1f, Q2a-2d and Q3a-3d

Due Date: Start of class on Friday, January 12, 2018

Integrals via u-Subs

Main strategy in integration

$\Rightarrow$ express an integral in a form that is **easily** recognizable

If we are lucky, the integral may already be in such a form:

(1a) $\int 6x^4 \; dx$

(1b) $\int -12e^x \; dx$

(1c) $\int \frac{5}{x} \; dx$

(1d) $\int \frac{1}{3\sqrt{x}} \; dx$

(1e) $\int \sin(x) \; dx$

(1f) $\int \left[x^7 - \frac{1}{2\sqrt{x^5}} + \sec^2(x)\right] \; dx$

Integrals via u-Subs

Unfortunately, much of the time the form will fit any **"table integral"**

New strategy in integration $\Rightarrow$ change an integral in a form that is **easily** recognizable

This is often difficult, but one simple case is when the integrand includes a function and its derivative!

Ex: $$\int 2x(x^2+12)^4\,dx = \frac{1}{5}(x^2+12)^5 + C$$

Essentially, here we are reverse engineering the **chain rule**!

Check: $$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{5}(x^2+12)^5 + C\right] = \frac{1}{5}\left[5(x^2+12)^4(2x)\right] = 2x(x^2+12)^4$$

This is why we look for a **function and its derivative**!

We make the process more recognizable by using a substitution

Integrals via u-Subs - Example

Use a substitution to replace the old variable (x) with a new variable (u)

Ex: $\int 2x(x^2+12)^4\ dx$ Integral wrt original variable, x

$= \int (x^2+12)^4\ [2xdx]$

Using the subtitution ① $u = x^2 + 12$ and ② $du = 2xdx$

$= \int (u)^4\ [du]$ Integral wrt new variable, u

$= \frac{1}{5}u^5 + C$ Evaluate the "table integral"

$=\frac{1}{5}(x^2+12)^5 + C$ Go back to the original variable using ①

Integrals via u-Subs - Example

Use this idea to evaluate the following integrals:

(2a) $\displaystyle\int x^3 \cos(x^4 - 3)\ dx$

(2b) $\displaystyle\int \frac{3dx}{\sqrt{x}(1+\sqrt{x})^2}$

(2c) $\displaystyle\int \sin(3x)e^{\cos(3x)}\ dx$

(2d) $\displaystyle\int \frac{1}{5x\log(x)}\ dx$

Definite Integrals via u-Subs

Definite integrals require extra care!

$$\int_{x_1=0}^{x_2=1} 2x(x^2+12)^4 \; dx \qquad \text{Note: The limits are values of } x$$

$$\int_{x_1=0}^{x_2=1} (u)^4 \; [du] \qquad \Leftarrow \text{WRONG: Limits should be values of } u$$

You must also change these values using the substituion

$$= \int_{u_1=12}^{u_2=13} (u)^4 \; [du] \qquad \text{CORRECT: New limits in } u$$

$$= \left[\tfrac{1}{5}(13)^5 + C\right] \text{-} \left[\tfrac{1}{5}(12)^5 + C\right] \qquad \text{Value of definite integral}$$

Compare the value by checking the integral wrt x!

Integrals via u-Subs - Example

Use this idea to evaluate the following integrals:

(3a) $\displaystyle\int_{-2}^{2} \frac{x}{\sqrt{4-x^2}}\,dx$

(3b) $\displaystyle\int_{0}^{1} \frac{e^x}{\cos^2(e^x)}\,dx$

(3c) $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{\sin(x)}{1+\cos^2(x)}\,dx$

(3d) $\displaystyle\int_{1}^{4} \frac{3dx}{\sqrt{x}(1+\sqrt{x})^2}$