MATH2260 - Calculus II for Science and Engineering

Assignment 3 - Due Start of Class on Monday, April 9, 2018

1 Fractal Geometries

Fractals are geometric constructions through an (infinite) iterated process. As a result, they are great objects to study using infinite series!

a) Starting with the unit square (side length of 1), consider the following process:

1. Divide each side into three equal subintervals (and hence 9 smaller squares)
2. Delete the "middle" square created in step 1
3. Repeat steps 1-2 for each newly created squares

Let R_n be the amount of area from the original square that has been removed by the n^{th} iteration. Determine an expression for R_n. Does it converge or diverge?
By summing up all of the R_n's, calculate the total area of original square that remains in the resulting fractal. Can you say anything about what remains in the fractal, if anything?

b) Starting with an equilateral triangle with side length 1, consider the following process:

1. Divide each of the sides into three equal segments
2. Along each side, construct a new equilateral triangle pointing outward so that the middle segments from step 1 are the base of each new triangle
3. Delete the middle segment of the original triangle sides
4. Repeat steps 1-3 for each side of the new structure

After n-iterations, determine the perimeter, P_n, and area, A_n, of the resulting object.
Using series, calculate the total perimeter ($P = \lim_{n\to\infty} P_n$) and enclosed area ($A = \lim_{n\to\infty} A_n$) for the resulting fractal.

2 Series Convergence

Determine the convergence/divergence properties of the following series. Justify your answer with an appropriate test/reasoning.

If the series is convergent and geometric or telescoping, calculate the associated sum.

i) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\log(n)}$ ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$ iii) $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$ iv) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} - \frac{n+2}{n+3}\right)$

v) $\sum_{n=1}^{\infty} \frac{2n^2 - 3n}{\sqrt{5 + n^5}}$ vi) $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ vii) $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ viii) $\sum_{n=4}^{\infty} \left(\frac{n}{n^2 + 4}\right)$ ix) $\sum_{n=1}^{\infty} \frac{9^n}{4 + 11^n}$

3 Miscellaneous

a) Find the value of c such that

$$\sum_{n=2}^{\infty} (1 + c)^{-n} = 2$$

b) Find the value of c such that

$$\sum_{n=0}^{\infty} e^{cn} = 10$$

c) Using geometric series, express the following repeating decimals as ratios of integers:

i) P=0.464646 ... ii) Q=1.5137137137 ... iii) R=0.2499999 ...

4 Power Series

Determine a) the center, b) the radius of convergence and c) the interval of convergence for each of the following power series.

i) $\sum_{n=1}^{\infty} \frac{(x+1)^{2n-2}}{(2n-1)!}$ ii) $\sum_{n=0}^{\infty} \frac{(x-4)^n}{n^n}$ iii) $\sum_{n=0}^{\infty} (-1)^n \frac{(x+1)^{2n-1}}{2n+1}$ iv) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$