**MATH2260 - Calculus II for Science and Engineering**

**Assignment 2 - Due Friday, March 2, 2018**

# 1 Shape Metrics

Evaluate the listed quantity:

a) Arclength of: $x = \frac{1}{3}\sqrt{y}(y-3), \quad \text{for } 1 \leq y \leq 9$

b) S. A. of revolution: $x = \frac{1}{3}(y^2+2)^{\frac{3}{2}}, \quad \text{for } 1 \leq y \leq 2 \text{ about the line } y = 0$

c) Volume of revolution: bounded by $x = 16 - y^2$ and $x = 0$ about the line $x = -1$

d) Volume of revolution: bounded by $x = 16 - y^2$ and $x = 0$ about the line $x = 17$

e) Arclength of: $y = \sqrt{1-x^2}, \quad \text{for } 0 \leq x \leq 1$

f) Volume of revolution: bounded by $y = 4x$ and $y = 2x^2$ about the $y$-axis

# 2 Volumes

Consider the region bounded by the curves $x + y = 1$ and $y = \sqrt{1-x^2}$.
Compute the volumes for the S.O.R. obtained by rotating about the:

a) the $y-$axis b) $y = 1$ c) $x = -2$

# 3 Semi-infinite

Consider the portion of the curve $y = 1/x$ for $x \geq 1$. By evaluating the appropriate integrals, show that:

a) The length of the curve is infinite

b) The surface area of the solid of revolution is infinite

c) The volume of the solid of revolution is finite, and find it's exact value

This is a shape that is impossible to draw or paint but can be filled with a finite volume!

# 4 Donut Integration

Consider the circle centred at $(3, 3)$ with radius 1:

a) Determine the volume of the S.O.R obtained by rotating about the line $y = 2$

b) Determine the surface area of the S.O.R obtained by rotating about the line $x = 0$

c) Determine the volume of the S.O.R obtained by rotating about the line $x = 1$

d) Determine the surface area of the S.O.R obtained by rotating about the line $y = -2$

# 5 Convergence of Integrals

For each of the following, determine if the integral is convergent or divergent. If convergent, find the value of the integral.

a) $\displaystyle\int_{-3}^{2} \frac{1}{w^4}\mathrm{d}w$ b) $\displaystyle\int_{-\infty}^{\infty} ye^{-y^2}\mathrm{d}y$ c) $\displaystyle\int_{-\infty}^{0} 2^r\mathrm{d}r$ d) $\displaystyle\int_{1}^{\infty} \frac{\log(x)}{x}\mathrm{d}x$ e) $\displaystyle\int_{-1}^{14} \frac{1}{\sqrt[4]{z+2}}\mathrm{d}z$

f) $\displaystyle\int_{1}^{\infty} \frac{1}{w^2-2w+10}\mathrm{d}w$ g) $\displaystyle\int_{1}^{2} \frac{3y^2+1}{y(y^2+1)}\mathrm{d}y$ h) $\displaystyle\int_{0}^{1} \frac{x}{(x+1)^4}\mathrm{d}x$ i) $\displaystyle\int_{-3}^{3} \frac{1}{x^2+x}\mathrm{d}x$

# 6 Differential Equations

Solve the following, utilizing the initial condition, where given:

a) $\dfrac{\mathrm{d}z}{\mathrm{d}u} - \dfrac{e^{u-z}}{1+e^u} = 0$ where $z(1) = 0$

b) $\dfrac{\mathrm{d}r}{\mathrm{d}x} = \dfrac{\log(x)}{rx}$ where $r(1) = 2$

c) $\dfrac{\mathrm{d}T}{\mathrm{d}x} = \sqrt{Tx}$ where $T(1) = 2$

d) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x\sqrt{1-y^2}$ where $y(0) = 0$

e) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{e^x}{(1+y)^2}$ where $y(0) = 0$