Improper Integrals

Recall that the definition of the integral is a limit!

$$\int_a^b f(t)\mathrm{d}t = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_i)\Delta x$$

So far, every integral we have evaluated has had a finite answer

There are two ways this can be fail to be true:

① : The limits of integration, a or b, could be infinite

② : The integrand $f(t)$ could be infinite for $a \leq x \leq b$

Improper Integrals - Infinite Limits

Example 1: Calculate $\int_1^\infty \frac{1}{x} dx$

Improper Integrals - Infinite Limits

Example 2: Calculate $\int_1^\infty \frac{1}{x^2} dx$

Improper Integrals - Infinite Limits

Example 3: Calculate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

Improper Integrals - Infinite Limits

Example 4: Calculate $\displaystyle\int_0^{\infty} \frac{9\arctan(x)}{1+x^2}dx$

Improper Integrals - Infinite Limits

Example 5: Calculate $\int_0^\infty \sin(x)dx$

Improper Integrals - Infinite Integrands

Example 6: Calculate $\int_0^1 \frac{1}{\sqrt{x}} dx$

Improper Integrals - Infinite Integrands

Example 7: Calculate $\displaystyle\int_0^3 \frac{1}{(x-1)^{2/3}}dx$

Improper Integrals - Infinite Integrands

Example 8: Calculate $\int_{-1}^{1} \frac{1}{x^2} dx$

Improper Integrals - Infinite Integrands

Example 9: Calculate $\int_0^{\frac{\pi}{4}} \tan(2x)dx$