MATH2260 - Calculus II for Science and Engineering

Assignment 3 - Due Tuesday November 7, 2017

1 Fractal Geometries

Fractals are geometric constructions through an (infinite) iterated process. As a result, they are great objects to study using infinite series!

a) Starting with the (closed) interval on the real line from 0 to 1, consider the following process:

1. Divide each interval into three equal subintervals
2. Delete the (open) middle subinterval(s) created in step 1
3. Repeat steps 1-2 for each newly created interval

Let R_n be the amount of the original interval, $0 \leq x \leq 1$, that has been removed by the n^{th} iteration. Determine an expression for R_n. Does it converge or diverge?

By summing up all of the R_n's, calculate the total length of original interval that remains in the resulting fractal. Can you say anything about the points that remain in the fractal, if any?

b) Starting with an equilateral triangle with side length 1, consider the following process:

1. Divide each of the sides into three equal segments
2. Along each side, construct a new equilateral triangle pointing outward so that the middle segments from step 1 are the base of each new triangle
3. Delete the middle segment of the original triangle sides
4. Repeat steps 1-3 for each side of the new structure

After n-iterations, determine the perimeter, P_n, and area, A_n, of the resulting object.

Using series, calculate the total perimeter ($P = \lim_{n\to\infty} P_n$) and enclosed area ($A = \lim_{n\to\infty} A_n$) for the resulting fractal.

2 Series Convergence

Determine the convergence/divergence properties of the following series. Justify your answer with an appropriate test/reasoning.

If the series is convergent and geometric or telescoping, calculate the associated sum.

i) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\log(n)}$ ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$ iii) $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$ iv) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} - \frac{n+2}{n+3}\right)$

v) $\sum_{n=0}^{\infty} e^{-n}$ vi) $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ vii) $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ viii) $\sum_{n=4}^{\infty} \left(\frac{n}{n^2+4}\right)$ ix) $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

3 Power Series

Determine a) the center, b) the radius of convergence and c) the interval of convergence for each of the following power series.

i) $\sum_{n=1}^{\infty} \frac{(x-3)^{2n-2}}{(2n-1)!}$ ii) $\sum_{n=0}^{\infty} \frac{(x+2)^n}{n^n}$ iii) $\sum_{n=0}^{\infty} (-1)^n \frac{(x+4)^{2n-1}}{2n+1}$ iv) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

4 Taylor Series

For the listed function, find the first non-zero five terms of the Taylor series generated by $f(x)$ at $x = a$.

i) $f(x) = \sqrt{x^3+1}$ at $x = 2$

ii) $f(x) = 3x^5 - 2x^4 + x^3 - 4x^2 - x + 5$ at $x = 1$

iii) $f(x) = \log(2+x)$ at $x = -1$

iv) $f(x) = e^{-x^2}$ at $x = 0$

v) $f(x) = \frac{1-\sqrt{1-4x^2}}{2x^2}$ at $x = 0$