Related Rates

Often we are interested in the rate of change of a function at a specific instant.

For simple functions, this is given by a direct derivative!

Ex: A pebble is dropped in a calm pond causing a circular ripple which grows outward. Determine how the area enclosed by the ripple is changing

a) as the ripple grows, b) when the ripple is 20cm.

Related Rates

Suppose however, that the rate we want to calculate depends on a different, but related quantity, which itself may also be changing.

Ex: As above, but we now measure that the radius grows at 3cm/s. Find the rate at which the area changes when the radius is 9cm.

Related Rates

Ex: If the area of a circle grows at a rate of 200π cm^2/s, find the rate of change of the radius when a) $r = 4$ and b) $r = 10$.

Related Rates

Ex: Suppose a 10 m ladder rests against a vertical wall. If the bottom of the ladder is being pulled at a rate of 1 m/s, find the rate at which the top of the ladder is moving down when the bottom is 8 m away from the wall.

Related Rates

Related Rates

Ex: John starts walking east from a point J at a speed of 2 m/s.
After 5 s, Kelly starts walking west, from a point 75 m due south of J, at a speed of 8 m/s.
How fast apart are the pair moving 9 s after the woman starts walking?

Related Rates

Related Rates

Ex: A water tank has the shape of an inverted circular cone with base radius 4m and height 2m. If water is pumped in at 2 m^3/min find the rate at which the level of water is rising when the water is 1.5 m deep.

Related Rates

Ex: A low-flying airplane's has a constant velocity of 200 m/s, flying at a constant altitude of 1000 m. Its flight is tracked by a radar dish on the ground.

Find the angular rate of change of the line connecting the radar dish to the plane when the plane is 1000 m to the right of the dish.

Related Rates

Linearizations

- If we know how $f(x)$ changes at $x = a$ we can say quite a lot about values of $f(x)$ close to $x = a$

- Using the information about $f'(x)$ we get a linear (first-order) approximation for $f(x)$

Approximation is exact at $x = a$ but may be 'bad' away from $x = a$

Linearizations

Ex: Find the linearization of $f(x) = \sin(x)$ at $x = 0$

Linearizations

Ex: Find the linearization of $f(x) = \sin(x)$ at $x = \pi/4$

Linearization - Applications

The utility of linearizations is threefold

1) Estimating $f(x)$ 2) Finding roots of $f(x)$ 3) Finding change in $f(x)$

In all cases we rely on the *linear* form to get our answer!

1) Estimating the value of $f(x)$

Linearization - Applications

2) Finding the root of $q(w)$

Linearization - Applications

Linearization - Applications

3) Estimating the change in of $r(z)$

Linearization - Applications