MATH2250 - Calculus I for Science and Engineering

Assignment 3 - Due Monday, April 9, 2018

1 Designing a Box

You are designing an open-top rectangular box. The width of the box must be twice as large as the length and a total volume of the box must be of 10 cm^3.

The material for the bottom of the box costs \$10 per cm^2 while material for the sides costs \$6 per cm^2. Determine the dimensions and overall cost for the cheapest such container.

2 Cylinders in Cones

A cylinder is inscribed in a right-circular cone in such a way that their bases are parallel.

If the height of the cone is 6m while the radius at it's base is 1m, determine the dimensions of the cylinder with the largest possible surface area.

3 Extreme Values

Determine all of the extreme values of the following functions on the interval indicated:

i) $f(x) = \sqrt{x^2 - 2x - 3}$ for $3 \leq x < \infty$ ii) $g(m) = m - 2\arctan(m)$ for $0 \leq m \leq 4$

iii) $r(t) = \frac{t^2}{4-t^2}$ for $-1 \leq t \leq \frac{1}{2}$ iv) $p(u) = \dfrac{u}{u^2 - u + 1}$ on $u \in [0, 3)$

4 Curve Sketching Lite

Use a set of axes and a reasonable scale to sketch the graph of a **single** function that satisfies **ALL** of the following conditions:

i) $f(0) = -2$ ii) $\lim\limits_{x \to 4^+} f(x) \to -\infty$ iii) $\lim\limits_{x \to 4^-} f(x) \to -\infty$ iv) $f(1)$ D. N. E.

v) $\lim\limits_{x \to -2^+} f(x) \to \infty$ vi) $\lim\limits_{x \to -2^-} f(x) \to -\infty$ vii) $\lim\limits_{x \to \infty} f(x) \to 0^+$ viii) $\lim\limits_{x \to -\infty} f(x) \to -2^-$

ix) $f'(x) > 0$ on $(-1, 1) \cup (4, 6)$ x) $f'(x) < 0$ on $(-\infty, -1) \cup (1, 4) \cup (6, \infty)$

xi) $f''(x) > 0$ on $(-2, 3) \cup (8, \infty)$ xii) $f''(x) < 0$ on $(-\infty, -2) \cup (3, 8)$

5 Curve Sketching

a) Use a set of axes and a reasonable scale to sketch the graph of

$$f(x) = \frac{1}{x^2 + 2x + 2}$$

Label all notable features of the graph (extreme values, inflection points, roots, asymptotes) as well as increasing/decreasing and C.U/C.D regions.

b) Use a set of axes and a reasonable scale to sketch the graph of

$$g(x) = e^{-x^2}$$

Label all notable features of the graph (extreme values, inflection points, roots, asymptotes) as well as increasing/decreasing and C.U/C.D regions.

6 Even More Limits

Evaluate the following, justifying your methods and solutions:

i) $\lim_{x\to 5} \dfrac{5x-2}{|5-x|}$ ii) $\lim_{s\to 0} 3s\cot(4s)$ iii) $\lim_{s\to -\infty} \dfrac{3s^3-1}{|8s^3+1|}$ iv) $\lim_{v\to\infty} v\tan\left(\dfrac{2}{v}\right)$

v) $\lim_{t\to 0} \dfrac{t\sin(t)}{\cos(t)-1}$ vi) $\lim_{s\to 0^+} (4s+1)^{\csc(s)}$ vii) $\lim_{m\to\infty} \dfrac{e^m + m^2}{e^m - m^3}$ viii) $\lim_{r\to\infty} r^2 e^{-2r}$

7 Linearization Bouquet

Consider the functions:

$$F(z) = z^2 - 2z + 1 \qquad G(z) = e^{-2z} \qquad H(z) = 1 + \log(1-2z)$$

Determine the linearization of each function at $a = 0$ and use the results to approximate the value of each function at $z = 1/4$.

Interpret the results, discussing the relative quality (which is best? which is worst?) of each approximation. Justify the relative size (over or under) of the approximation compared to the true value of the respective function.