MATH2250 - Calculus I for Science and Engineering

Assignment 2 - Due Friday, March 2, 2018

1 Derivatives from First Principles

Given the function $h(q) = \dfrac{15}{\sqrt{2q-1}}$, **from first principes** calculate:

a) $h'(1)$ b) $h'(5)$ c) $\left.\dfrac{dh}{dq}\right|_{q=13}$ and d) $\dfrac{dh}{dq}$

Hint: You can use your result from part d) to check your answer for parts a)-c). However, you still **must** do parts all parts from first principles!

2 Derivatives

Calculate the following derivatives. Simplify your final results, where applicable.

i) $\dfrac{\mathrm{d}^2}{\mathrm{dx}^2}[\tan(e^x)]$ ii) $\left.\dfrac{\mathrm{dr}}{\mathrm{dm}}\right|_{m=0}$ where $r(m) = \sin^2(m)\arccos(1-m)$

iii) $\dfrac{\mathrm{dg}}{\mathrm{dz}}$ where $g(z) = \arctan\left(\dfrac{1}{z}\right)$ iv) $\dfrac{\mathrm{dw}}{\mathrm{ds}}$ where $\dfrac{4\sin^2(s)}{w^2+1} = 2e^{-2w}$ at $w(\frac{\pi}{4}) = 0$

v) $\dfrac{\mathrm{dp}}{\mathrm{dt}}$ where $p(t) = [\arccos(t)]^{\log(t)}$ vi) $\dfrac{\mathrm{d}}{\mathrm{dq}}\left[\sqrt{\dfrac{\cot(q)\arctan(2^q)}{q^2-1}}\right]$

vii) $\dfrac{\mathrm{d}^{10}\mathrm{g}}{\mathrm{dx}^{10}}$ where $g(x) = \dfrac{1}{1-x}$ viii) $\dfrac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}$ where $xe^y = 3$ ix) $\dfrac{\mathrm{d}^9\mathrm{W}}{\mathrm{dz}^9}$ where $W(z) = \log(z)$

3 More Limits

Evaluate the following limits if they exist, justifying your answers. If the limit does not exist, explain why.

i) $\displaystyle\lim_{x\to 3} \frac{|x^2-9|}{|x|-3}$ ii) $\displaystyle\lim_{s\to 0} \sin(3s)\cot(2s)$ iii) $\displaystyle\lim_{r\to 9} \frac{r-9}{\sqrt{r}-3}$ iv) $\displaystyle\lim_{v\to\infty} \frac{\arctan(2v)}{1-v^2}$

4 Tangent Lines

a) Determine the equation of the tangent lines in reference to $p(x) = \sqrt{\tan(\pi x) + 8}$ at:

a) $x = 0$ and b) $x = \dfrac{1}{4}$

b) Determine the equation of the tangent line to $Q(m) = \log(\cos(m))$ at:

a) $m = 0$ b) $m = \dfrac{\pi}{4}$ and c) $m = -\dfrac{\pi}{4}$

c) Determine the equation of the tangent line to $R(s) = \cosh(s) = \dfrac{e^s + e^{-s}}{2}$ at:

a) $s = 0$ b) $s = \log(2)$ and c) $s = -\log(3)$

5 Continuity

Determine the value(s) of k which ensure that $f(x)$ is continuous **for all** x.

$$f(x) = \begin{cases} k^2 \arccos(x) & x \leq 0 \\ \dfrac{\sin(4\pi x)}{\pi x} & 0 < x \leq 1 \\ k + \dfrac{x^2 - 1}{x - 1} & x > 1 \end{cases}$$

6 Table Derivatives

Below is a list of properties of the functions $f(x)$ and $g(x)$.

x	-4	-1	0	2	4	5
$f(x)$	-log(4)	3	-1	-6	3	1
$f'(x)$	5	-1/3	-1	2	1	-2
$g(x)$	-1	6	1/2	1	-2	0
$g'(x)$	4	-4	0	-1	0	log(2)

Using this information, evaluate the listed quantities, simplifying fully.

i) $K'(-1)$ where $K(x) = f(x) \cdot g(x)$ ii) $L'(4)$ where $L(x) = \left(\frac{f(x)}{g(x)}\right)^2$

iii) $Q'(-4)$ where $Q(x) = [f(g(x))]^3$ iv) $R'(0)$ where $R(x) = [g(x)]^{f(x)}$