Functions

Calculus - the study of change, as related to functions

Elementary functions will be the building blocks of our study

Polynomial

trigonometric

exponential

radical and

logarithmic functions

Based on these functions and their properties, we can create more complicated **composite** functions

Fractional and

branch functions

Functions - Review

If calculus is a new language, functions are its words!

Must be '***fluent***' in elementary functions

1) polynomials

General form: $f(x) = a_0 + a_1x + a_2x^2 \ldots a_nx^n$

Ex: $f(x) = 3x - 5$ (linear), $g(t) = -t^2 + 2t + 5$ (quadratic)

$R(s) = 4$ (constant), $F(x) = 3x^7 - 9x^4t + 5x$ (7th order)

Domain: $D_f = \{x \in \mathbb{R}\}$

For constant, linear and parabolic functions, you must know:

- how to graph the function
- how to find its roots

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2) trigonometric functions $\Rightarrow f(x) = \sin(x)$ and $g(x) = \cos(x)$

Domain: $D_f = \{x \in \mathbb{R}\}$

Range: $R_f = \{y \in \mathbb{R} | -1 \leq y \leq 1\}$

For $\sin(x)$ and $\cos(x)$ you must know:

- the "ratio" definition from right triangles
- how to graph them
- how to evaluate special angles - $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

For composite trig functions, rewrite using $\sin(x)$ and $\cos(x)$

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3) exponentials

General form: $f(x) = Ka^x$ - exponential with base $a > 0$

Ex: $f(x) = 3^x$ (base 3), $g(t) = \left(\frac{2}{5}\right)^t$ (base $\frac{2}{5}$)

Domain: $D_f = \{x \in \mathbb{R}\}$

Range: $R_f = \{y \in \mathbb{R} | 0 < y < \infty\}$

You must know:

- how to graph the cases $0 < a < 1$ and $a > 1$
- the exponent laws

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4) radicals

General form: $f(x) = K\sqrt[n]{x} = Kx^{1/n}$ - n-th root, integer $n \geq 1$

Ex: $f(x) = \sqrt{x}$ (square root), $g(t) = 3\sqrt[13]{t} = 3x^{1/13}$ (13th root)

Domain: Depends on the parity of n

n odd: $D_f = \{x \in \mathbb{R}\}$

n even: $D_f = \{x \in \mathbb{R} | 0 \leq x < \infty\}$

Range: Depends on the parity of n

n odd: $R_f = \{y \in \mathbb{R}\}$

n even: $R_f = \{y \in \mathbb{R} | 0 \leq y < \infty\}$

You must know:

- how to graph the cases $n = 2$ and $n = 3$
- how these functions relate to polynomials

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5) logarithms

General form: $f(x) = K \log_a x$ - logarithm with base $a > 0$

Ex: $f(x) = \log_3(x)$ (base 3), $g(t) = \log_{1/2}(t)$ (base $\frac{1}{2}$)

$F(x) = \log_e(x) = \log(x) = \ln(x)$ (base $e = 2.718...$, natural log)

Domain: $D_f = \{x \in \mathbb{R} | 0 < x \leq \infty\}$

Range: $R_f = \{y \in \mathbb{R}\}$

You must know:

- how to graph the cases $0 < a < 1$ and $a > 1$
- the logarithm laws
- how they relate to exponentials

Composite Functions

Elementary transformations can shift/stretch basic functions

Track changes to domain and range

Otherwise, we can combine the elementary functions to obtain more complicated ones.

6) fractional

General form: $f(x) = \dfrac{P(x)}{Q(x)}$ - where $P(x)$ and $Q(x)$ are of 1) - 5)

Ex: $f(x) = \dfrac{x+3}{x^2-1}$, $g(t) = \dfrac{\sin(t)}{e^{2t}+\log(t)}$, $H(r) = \dfrac{\sqrt{r-4}}{r^3+2\tan(r)}$

You must know:

- properties of the function will depend on $P(x)$ and $Q(x)$
- domain restricted by $Q(x) \neq 0$

Branch Functions

7) piecewise or branch functions

General form: $f(x) = \begin{cases} f_1(x) & \text{Condition 1} \\ f_2(x) & \text{Condition 2} \end{cases}$

Ex: $f(x) = \begin{cases} x^2 + 4 & \text{if } x \neq 2 \\ \sin(3x) & \text{if } x = 2 \end{cases}$, $g(t) = \begin{cases} \log_5(t) & \text{if } t \leq 4 \\ 7^t - \sqrt{t} & \text{if } x > 4 \end{cases}$

You must know:

- how to read conditions to pick the correct branch
- domain restricted by each branch
- may have more than 2 branches/conditions
- how to write absolute values as branch functions