# Integrals - Motivation

When we looked at a function's rate of change

If $f(x)$ is linear, the answer is easy $\rightarrow$ slope

If $f(x)$ is non-linear, we had to work hard $\rightarrow$ limits $\rightarrow$ derivative

A related question is the area 'under' $f(x)$ (but above the $x-$axis)

If $f(x)$ is constant, we can resort to some geometry

Clearly the answer depends on the two 'bounding' points!

Notation:

# Integrals - Linear Functions

If $f(x)$ is linear, we can still use geometry

Also for $f(x) = \sqrt{a^2 - x^2}$

# Example

Ex: Approximate the area under $f(x) = 1 - x$ between $a = 0$ and $b = 1$ using the approximation a) with 5 rectangles, b) with 10 rectangles

# Example - Right Endpoints

# Integrals - Approximation

By increasing the number of rectangles, we get a better approximation!

Define $I_n$ - the approximated area (using $n$ rectangles)

Width of each rectangle becomes $\Delta x = \dfrac{b-a}{n}$

Height of each rectangle is $f(c_i)$ for some $c_i$ in the $i$-th sub-interval

# Integrals - Riemann Sum

We can imagine taking the number of approximating rectangles to be extremely large

The resulting quantity is called the **Riemann sum**

NOTE: We arbitrarily chose $c_i$ as the left end-point of the $i$-th subinterval

In the limiting case, we can choose any point in each sub-interval!

The method of Riemann sums is completely general

Can be used with general $f(x)$ (not just linear)

# Riemann Sum - Example

Ex: Approximate the area under $f(x) = 1 - x^2$ between $a = 0$ and $b = 1$ by using the Riemann sum approximation in the large $n$ limit

# Riemann Sum - Example