Indefinite Integrals

Recall that $f'(a)$ is just a number (slope of the tangent to $f(x)$ at $x = a$)

In a similar way, the definite integral of $f(x)$ between $x = a$ and $x = b$ is just a number

Net area bounded by $f(x)$, the x-axis, $x = a$ and $x = b$

Indefinite Integrals

To get the derivative function, $f'(x)$, we left the point unspecified!

We get the *integral function* by leaving the limits unspecified

The resulting construct is known as the **indefinite integral**

Fundamental Theorem of Calculus I

If $F(x) = \int_a^x f(t)\,\mathrm{d}t$, then we say that $F(x)$ is the antiderivative of $f(x)$

The **Fundamental Theorem of Calculus** (FTC) relates these two objects from integral and differential calculus

1) Suppose $f(x)$ is a continuous function on $a \leq x \leq b$.

$$\text{If } F(x) = \int_a^x f(t)\mathrm{d}t,$$
$$\text{then } F'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f(t)\,\mathrm{d}t = f(x)$$

This gives us a relationship between the indefinite integral and the derivative funciton!

Implication: **integrals are the inverses of derivatives**!

FTC I - Example

Ex: Use FTC I to calculate $\frac{\mathrm{d}f}{\mathrm{d}x}$ in each case:

a) $f(x) = \int_a^x (t^3 + 1)\mathrm{d}t$

b) $f(x) = \int_{4x}^3 \sin(r^2)\mathrm{d}r$

c) $f(x) = \int_0^{\sin(x)} \frac{\mathrm{d}t}{\sqrt{1 - t^2}}$

Fundamental Theorem of Calculus II

If $F(x) = \int_a^x f(t)\,\mathrm{d}t$, then we say that $F(x)$ is the antiderivative of $f(x)$

The **Fundamental Theorem of Calculus** (FTC) relates these two objects from integral and differential calculus

2) Suppose $f(x)$ is a continuous function on $a \leq x \leq b$ and $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(t)\mathrm{d}t = \int_a^b \frac{\mathrm{d}F}{\mathrm{d}t}\mathrm{d}t = F(b) - F(a)$$

This gives us a relationship between the definite integral and the derivative at a point!

Implication: **integrals are the inverses of derivatives**!

FTC II - Example

Ex: Use FTC II to evaluate the following integrals:

a) $\displaystyle\int_0^3 (t^2 + 1)\mathrm{d}t$

FTC II - Example

Ex: Use FTC II to evaluate the following integrals:

b) $\displaystyle\int_{\pi}^{\frac{\pi}{4}} \sin(r)\mathrm{d}r$

FTC II - Example

Ex: Use FTC II to evaluate the following integrals:

c) $\displaystyle\int_2^8 \frac{\mathrm{d}t}{1+t}$

Integrals

We do integrals by *recognizing* the integrand as the derivative of a known function

Ex: $\int (3t - 15)\,\mathrm{d}t$

Ex: $\int_1^9 \frac{1}{q^2}\mathrm{d}q$

Ex: $\int \frac{\mathrm{d}s}{s}$

Integrals

We do integrals by *recognizing* the integrand as the derivative of a known function

$$\int c f(x)\,dx = c\int f(x)\,dx \qquad \int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

Integrals - Substitution

What about other integrals?

Ex: $\displaystyle\int \tan\theta \mathrm{d}\theta$

Integrals - Substitution

What about other integrals?

Ex: $\displaystyle\int \frac{t}{1-t^2}\,\mathrm{d}t$

Integrals - Substitution

What about other integrals?

Ex: $\displaystyle\int \sqrt{2t-9}\,\mathrm{d}t$

Integrals - Substitution

What about other integrals?

Ex: $\displaystyle\int x\sqrt{2x-9}\,\mathrm{d}x$

Integrals - Substitution

What about other integrals?

Ex: $\displaystyle\int \frac{1}{e^r + e^{-r}} \mathrm{d}r$