

(Study)

Ch. 6-10 - Sample Questions

- 1.) A certain factory produces shelves for closets. The length of these shelves has a normal distribution, with expectation 170 centimeters^(cm) and standard deviation 0.625 cm. Shelves that are larger than 171 cm or smaller than 168 cm cannot be used directly to make a certain type of closet. What percentage of the shelves can be used directly for these closets?
- 2.) A random variable X has a normal distribution with parameters μ_x and $\sigma_x^2 = 4$. It is known that $P(X \leq 4) = .9772$. Determine the value of μ .
- 3.) A random variable X has a normal distribution with mean equal to 20 and variance equal to 4. Suppose that $P(X \geq a) = 0.67$. Determine a .
- 4.) A student asks 36 individuals whether they want to purchase the 1992 calendar of his fraternity. The profits will be given to charity. The random variable Y , the number of individuals who decide to buy the calendar, has a binomial distribution with $p = .5$. Calculate $P(Y \geq 20)$ using an appropriate approximation.

5.) ~~7~~ A ^{simple} random sample from a population of size ~~45~~ 45 results in the following observations:

13, 17, 9, 15 and 11

Estimate ~~Determine~~ the ~~sample~~ variance of the sample mean.

6.) ~~8~~ A simple random sample of size $n=16$ is obtained from an infinitely large population that follows a normal distribution with expectation $\mu=10$ and variance $\sigma^2=100$. Determine the probability that the sample mean exceeds 4.

7.) In the basement of an old building on campus there is a coffin with 100 jewels. These jewels contain, on average, 1 ounce ($\mu=1$) pure gold per jewel, with a standard deviation $\sigma=.3$. A thief takes a sample without replacement of 36 jewels from the coffin. (Assume that the sample is random ~~in~~ in all other respects.) Determine by approximation the probability that the ^{stolen} jewels ~~in the sample~~ contain, on average, less than 1.11 ounce of gold per jewel.

8) ~~8~~ Consider a random sample of size n from an unknown distribution with expectation μ_x and variance σ_x^2 . Which of the following statements regarding the distribution of \bar{X} is true according to the Central Limit Theorem:

- The distribution of \bar{X} is approximately normal, because you may assume that the underlying distribution is normal.
- The dist'n of \bar{X} is approximately normal with expectation μ_x and variance $\sigma_x^2/(n-1)$
- The distribution of \bar{X} is approximately normal if n is large
- None of the above are true because the dist'n from which the sample is drawn is unknown.

9.) A ^{simple} random sample of size 10 ^(from an infinite population) is taken from a normal dist'n with unknown expectation μ_x and variance σ_x^2 , resulting in $\bar{X} = 83$ and $S_x^2 = 15$. Construct a 90% confidence interval for μ_x .

10) Suppose that $X_1, X_2, \dots, X_{100} \stackrel{iid}{\sim} N(5, 16)$

- Find the mean and variance of \bar{X} , the sample mean
- Find the distribution of \bar{X} .
- Find the probability that \bar{X} is greater than 6.

10 (continued) Let $Y = X_1 + X_2 - 2X_3$

- d.) Find the mean and variance of Y
- e.) Find the distribution of Y
- f.) Find $P(-1 < Y < 1)$

11) Suppose that a random sample of 16 observations from a normal distribution yields a sample mean and a sample variance of 22 and 9, respectively. We are interested in the mean of the distribution from which the sample was drawn.

- a.) What is the point estimate of the mean?
- b.) Construct a 95% confidence interval for the mean under the assumption that the variance is known to equal 10.
- c.) Construct a 95% C.I. for the mean assuming the variance is unknown.
- d.) Construct a 90% CI for the mean assuming the variance is unknown.

12.) Let X_1, X_2, \dots, X_{100} be a ~~sample~~ ^{simple random sample} ~~of size~~ a SRS from a normal distribution w/ mean 0 and variance 100.

- a.) Find the probability that $X_1 + X_2 + \dots + X_{100}$ exceeds 100
- b.) Find the probability that more than 30 of the X_i 's exceed 10

Questions

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13) A British restaurant chain & presently uses cod for "fish and chips," the fish traditionally used in this dish. However, whiting is a cheaper and more plentiful fish. Management has decided to ~~with~~ conduct a taste-test involving a random sample of $n=265$ consumers to determine if p , the population proportion who prefer whiting to cod, is equal to .5 which would indicate that the fish are interchangeable.

Suppose that 144 of 265 consumers prefer whiting.

a) Test $H_0: p = .5$ versus a two-tailed alternative using a significance level of .05.

b) Form an ^{approx.} 90% confidence interval for p .