STAT 8250 — Applied Multivariate Methods
Lab – Due: Monday, Oct. 2

Example 5.2 in the text describes a study in which perspiration from 20 healthy females was analyzed. Three components, x_1 =sweat rate, x_2 =sodium content, and x_3 =potassium content, were measured and are presented below.

Individual	x_1	x_2	x_3
1	3.7	48.5	9.3
2	5.7	65.1	8.0
3	3.8	47.2	10.9
4	3.2	53.2	12.0
5	3.1	55.5	9.7
6	4.6	36.1	7.9
7	2.4	24.8	14.0
8	7.2	33.1	7.6
9	6.7	47.4	8.5
10	5.4	54.1	11.3
11	3.9	36.9	12.7
12	4.5	58.8	12.3
13	3.5	27.8	9.8
14	4.5	40.2	8.4
15	1.5	13.5	10.1
16	8.5	56.4	7.1
17	4.5	71.6	8.2
18	6.5	52.8	10.9
19	4.1	44.1	11.2
20	5.5	40.9	9.4

Suppose that we are interested in testing the hypothesis that the mean vector, $\boldsymbol{\mu} = \mathrm{E}[\mathbf{x}] = \mathrm{E}[(x_1, x_2, x_3)'] = (\mu_1, \mu_2, \mu_3)'$, is equal to the vector $\boldsymbol{\mu}_0 = (4, 50, 10)'$. That is, we want to test

$$H_0 : \boldsymbol{\mu} = \begin{pmatrix} 4 \\ 50 \\ 10 \end{pmatrix} \quad \text{versus} \quad H_1 : \boldsymbol{\mu} \neq \begin{pmatrix} 4 \\ 50 \\ 10 \end{pmatrix}$$

Copy the file sweat1.sas from the public data directory to your home directory. Run sweat1.sas and look at the program and output. Sweat1.sas performs Hotelling's T^2 test.

1. What is the value of your test statistic, T^2?

2. What is the critical value for this test, based on a significance level of $\alpha = 0.05$?

3. What conclusion do you draw from this test?

Consider once again the cork data in cork.dat which you should already have from the first lab. Recall that the cork data consist of the weights of cork from borings into 28 trees from each direction. The data consist of four variables, N, E, S, W which are cork weights measured on each of 28 trees.

Suppose that we are interested in testing the null hypothesis that the mean cork weights are the same in each direction. We can express this hypothesis as $H_0 : \mu_N = \mu_E = \mu_S = \mu_W$. If we define

$$
\begin{aligned}
x_1 &= N - E \quad \text{(diff. between north and east measurements)} \\
x_2 &= E - S \quad \text{(diff. between east and south measurements)} \\
x_3 &= S - W \quad \text{(diff. between south and west measurements)}
\end{aligned}
$$

then we can re-express our null hypothesis equivalently as $H_0 : \boldsymbol{\mu} = (0, 0, 0)'$ where $\mu_1 = \mathrm{E}(x_1)$, $\mu_2 = \mathrm{E}(x_2)$, and $\mu_3 = \mathrm{E}(x_3)$.

4. For the cork data write a SAS program to compute a test statistic to test $H_0 : \boldsymbol{\mu} = (0, 0, 0)'$ versus $H_1 : \boldsymbol{\mu} \neq (0, 0, 0)'$. What is the value of your test statistic?

5. Using a significance level of 0.10, what is the critical value for your test?

6. Are the mean cork weights the same in each direction?