STAT 6200 Homework #6 Solutions

Ch. 11

(10.a) $n_1 = 156$, $\bar{x}_1 = 54.8$, $s_1 = 28.1$ ← population mean μ_1

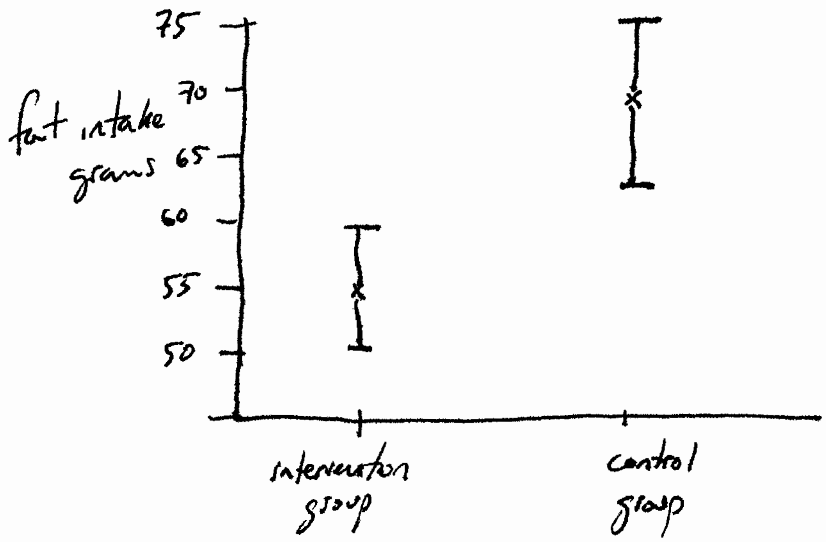
$n_2 = 148$, $\bar{x}_2 = 69.5$, $s_2 = 34.7$ ← pop. mean μ_2

95% CI for μ_1: $\bar{x}_1 \pm t_{1-.05/2}(n_1 - 1)\ s_1/\sqrt{n_1}$

$$= 54.8 \pm \underbrace{t_{.975}(155)}_{=1.97539}\ 28.1/\sqrt{156} = (50.36, 59.24)$$

95% CI for μ_2: $\bar{x}_2 \pm t_{.975}(n_2 - 1)\ s_2/\sqrt{n_2}$

$$= 69.5 \pm \underbrace{t_{.975}(147)}_{=1.97623}\ 34.7/\sqrt{148} = (63.86, 75.14)$$



← Since they don't overlap, we would expect that the two population means are not equal.

10 b. First test $H_0: \sigma_1^2 = \sigma_2^2$ using an F test

$$F = \frac{s_2^2 \leftarrow \text{larger}}{s_1^2} = \frac{(34.7)^2}{(28.1)^2} = 1.52$$

The critical value is $F_{1-.05}(n_2-1, n_1-1) = F_{.95}(147, 155) = 1.30748$
so we reject $H_0: \sigma_1^2 = \sigma_2^2$
Therefore, use the case 3 t-test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{54.8 - 69.5}{\sqrt{\frac{28.1^2}{156} + \frac{34.7^2}{148}}} = -4.0464$$

$p = 2P(t(\nu) > |t|)$ where $\nu = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$

$= 2P(t(282) > 4.0464)$

$= 2(.0000336) = .0000672$

$< \alpha = .05$

$$= \frac{\left(\frac{28.1^2}{156} + \frac{34.7^2}{148}\right)^2}{\frac{(28.1^2/156)^2}{155} + \frac{(34.7^2/148)^2}{147}}$$

$= 282.9 \leftarrow$ round down to 282

so reject $H_0: \mu_1 = \mu_2$
and conclude that the population mean fat intake among the husbands in the intervention group is not the same as the population mean fat intake for the husbands in the control group.

10 c) 95% CI for $\mu_1 - \mu_2$:

$$\bar{X}_1 - \bar{X}_2 \pm t_{1-\frac{.05}{2}}(\nu)\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$\underbrace{(54.8 - 69.5)}_{= -14.7} \pm \underbrace{t_{.975}(282)}_{= 1.96841}\sqrt{\frac{28.1^2}{156} + \frac{34.7^2}{148}}$$

$$= (-21.85, -7.55)$$

d.) $\bar{X}_1 = 172.5$, $S_1 = 68.8$, $n_1 = 156$
$\bar{X}_2 = 185.5$, $S_2 = 69.0$, $n_2 = 148$

$$F = \frac{(69.0)^2}{(68.8)^2} = 1.0058, \quad F_{1-.05}(147, 155) = 1.30748$$

So since $F < 1.30748$, we fail to reject $H_0: \sigma_1^2 = \sigma_2^2$
So, use case 2 t-test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad \text{where } S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$$

$$= \frac{155(68.8^2) + 147(69.0^2)}{156 + 148 - 2}$$

$$= \frac{172.5 - 185.5}{\sqrt{4746.85\left(\frac{1}{156} + \frac{1}{148}\right)}} \qquad = 4746.85$$

$$= -1.64 \qquad p = 2P(t(n_1+n_2-2) > |t|) = 2P(t(302) > 1.64)$$
$$= 2(.05057) = .1011$$

So fail to reject $H_0: \mu_1 = \mu_2$. Insufficient evidence to conclude that the population mean carb intake differs.

11. a) $H_0: \mu_1 = \mu_2$ vs $H_A: \mu_1 < \mu_2$ where μ_1 = population mean carboxyhemoglobin level for non-smokers and μ_2 = " " " " " smokers.

b) Use case 3:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1.3 - 4.1}{\sqrt{\frac{1.3^2}{121} + \frac{2.0^2}{75}}} = \cancel{10} -10.79$$

p value = $\cancel{P(t(\nu) < t)}$ $P(t(\nu) < -10.79)$

where

$$\nu = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}} = \frac{\left(\frac{1.3^2}{121} + \frac{2.0^2}{75}\right)^2}{\frac{\left(\frac{1.3^2}{121}\right)^2}{120} + \frac{\left(\frac{2.0^2}{75}\right)^2}{74}}$$

$= 113.05$ round down to 113

$\Rightarrow p = P(t(113) < -10.79) = .0000000$ (to that many decimal places)

So reject $H_0: \mu_1 = \mu_2$ and conclude that $\mu_1 < \mu_2$

13 a) Descriptives for bed80 and bed86 (see Minitab project hwk6-13.mpj):

	n	mean	SD	$SE(\bar{x})$	Min	Q1	Q2	Q3	Max
bed80	51	4.557	1.013	.142	2.7	3.7	4.5	5.1	7.4
bed86	51	4.233	1.107	.155	2.4	3.4	4.2	5.0	7.7

13b.) See hwk6-13.mpj (Minitab project). Here we do the paired t-test to yield
& c) the following results:
& d)

$t = 6.87$, $p = .000$ so, at $\alpha = .05$, we reject $H_0: \mu_1 = \mu_2$
where μ_1 = pop. mean # of beds in 1980
μ_2 = " " " " " " 1986

~~So~~ and conclude that $\mu_1 \neq \mu_2$
In particular, μ_1 appears to be lower than μ_2 (fewer beds in 1980)

The two-sample t-test (which is wrong here) yields the following results

First, an F-test gives $F = 1.1956$, $p = .530$
so fail to reject $H_0: \sigma_1^2 = \sigma_2^2$, so use case 2

$t = 1.54$, $p = .127$ so we fail to reject H_0 according to this test. This result shows that if we do the wrong test we can get quite different results.

e) From minitab: $(.2290, .4181)$

4. Because in the hypothesis test, the standard error is based on the distribution of $\hat{p}_1 - \hat{p}_2$ under the null hypothesis that $p_1 = p_2$. For a confidence interval, the standard error is based on the distribution of $\hat{p}_1 - \hat{p}_2$ in general (not necessarily under the null hypothesis).

7. $n = 27$ infants. $X = 15$ w/ mothers who smoked

a) $\hat{p} = \frac{X}{n} = \frac{15}{27} = .5556$

95% CI: $\hat{p} \pm z_{1-\frac{.05}{2}} \sqrt{\hat{p}(1-\hat{p})/n}$

$= .5556 \pm 1.96 \sqrt{(.5556)(1-.5556)/27} = (.3681, .7430)$

b) $p_0 = .328$. $H_0: p = p_0$ vs $H_A: p \neq p_0$ where $p_0 = .328$
+c)

d) $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \dfrac{.5556 - .328}{\sqrt{.328(1-.328)/27}} = 2.52$

p-value $= 2P(Z > 2.52) = 2(.006) = .012$

e) Fail to reject $H_0: p = p_0$ at $\alpha = .01$. There is insufficient evidence to conclude that the proportion of children w/ oral cleft whose mothers smoked during pregnancy is different than the corresponding population proportion (.328) for other malformations

f.) $p_1 = .25$ $\quad n = \dfrac{p_1(1-p_1)\left\{z_{1-\beta} + z_{1-\alpha/2}\sqrt{\dfrac{p_0(1-p_0)}{p_1(1-p_1)}}\right\}^2}{(p_0 - p_1)^2}$

$z_{1-\beta} = z_{.90} = 1.28155 \quad z_{1-\alpha/2} = z_{1-\frac{.01}{2}} = z_{.995} = 2.57583$

So, $n = \dfrac{(.25)(.75)\left\{1.2855 + 2.57583\sqrt{\dfrac{.328(1-.328)}{.25(.75)}}\right\}^2}{(.328-.25)^2}$

$= 511.60 \approx 512$

8. $n = 488$, $x = 473$

a) $\hat{p} = 473/488 = .9693$

b.) $\hat{p} \pm \underbrace{z_{1-\alpha/2}}_{=1.96} \sqrt{\hat{p}(1-\hat{p})/n} = .9693 \pm 1.96\sqrt{.9693(1-.9693)/488}$
$= (.9539, .9846)$

c) We can be 95% confident that the proportion of terminated pregnancies is between 95.39% and 98.46%. (That is, if repeated many times, the procedure that led to this interval would cover the true proportion 95% of the time)

d.) $\hat{p} \pm \underbrace{z_{1-\frac{.10}{2}}}_{=1.645} \sqrt{\hat{p}(1-\hat{p})/n} = (.9564, .9821)$

e) The 90% CI is narrower.

10. $x_1 = 11$, $n_1 = 114$, $x_2 = 7$, $n_2 = 96$

a) $\hat{p}_1 = \frac{x_1}{n_1} = .09649$, $\hat{p}_2 = \frac{x_2}{n_2} = \frac{7}{96} = .07292$

$\hat{p}_1 - \hat{p}_2 = \frac{11}{114} - \frac{7}{96} = .02357$

b.) $$\hat{p}_1 - \hat{p}_2 \pm \overbrace{z_{1-\alpha/2}}^{=1.96} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$= \frac{11}{114} - \frac{7}{96} \pm 1.96 \sqrt{\frac{\left(\frac{11}{114}\right)\left(1-\frac{11}{114}\right)}{114} + \frac{\frac{7}{96}\left(1-\frac{7}{96}\right)}{96}}$$

$$= (-.05154, .09869)$$

c) $$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$ where $\hat{p} = \frac{11+7}{114+96} = .08571$

$= .61$ ~~p-value~~

two-sided p-value $= 2P(z > .61) = 2(.271) = .543$

Fail to reject $H_0: p_1 = p_2$

d.) It does not appear that the physicians' advice is effective in encouraging patients to quit smoking.

12. $\hat{p}_1 = .247, n_1 = 97 \Rightarrow X_1 = n_1\hat{p}_1 = 97(.247) = 24$
$\hat{p}_2 = .174, n_2 = 161 \Rightarrow X_2 = n_2\hat{p}_2 = 161(.174) = 28$

a) $$\hat{p} = \frac{24+28}{97+161} = .2016$$

b.) $$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left\{\frac{1}{n_1}+\frac{1}{n_2}\right\}}} = 1.43$$

c) $p = 2P(z > 1.43) = 2(.076) = .154$ Fail to reject $H_0: p_1 = p_2$
Insufficient evidence to conclude that population proportions are different.

d) $\hat{p}_1 - \hat{p}_2 \pm \underbrace{z_{1-\frac{.05}{2}}}_{=1.96} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

$= (-.03042, .1774)$

Exact results

7a) $(.3533, .7452)$ ← exact vs $(.3681, .7430)$ ← approximate

exact is slightly wider

d.) Exact p-value = .022 approximate p-value = .012

+e). At $\alpha = .01$ we fail to reject w/ either method, but exact p-value is bigger