STAT 8620 — Categorical Data Analysis and Generalized Linear Models
Homework 2 – Due Tuesday, Sept. 18, 2012
SHOW ALL WORK

- Homework is due by 4:30 on the due date specified above. You may turn it in to me during class, slip it under my door or send it to me via e-mail. I will post homework solutions shortly after all homeworks have been collected. **No late homeworks will be accepted without permission granted prior to the due date.**

- Use only standard (8.5×11 inch) paper and use only one side of each sheet.

- Homework should show enough detail so that the reader can clearly understand the procedures of the solutions. This is **absolutely essential** for you to receive full credit for your answer.

- Problems should appear in the order that they were assigned.

Assignment:

1. For each of the following densities for a random variable Y, show that Y or some transformation of Y has an exponential family distribution. Derive the mean and variance of the exponential family distributed quantity in each case using the mean and variance formulas that hold in general within the E.D. family.

   a. The inverse Gaussian distribution with density

   $$f_y(y;\mu,\lambda) = (2\pi y^3/\lambda)^{-1/2}\exp\{\frac{-\lambda}{2\mu^2}(y-\mu)^2/y\}, \quad y,\lambda,\mu > 0.$$

   b. The Pareto distribution with density

   $$f_y(y;\theta) = \theta a^\theta / y^{(\theta+1)}, \quad y > a, \theta > 0, a > 0$$

2. Determine whether the following distributions belong to the ED family. If not, is there a transformation of the random variable Y which does have an ED distribution?

   a. The extreme value (Gumbel) distribution with density

   $$f_Y(y;\theta,\phi) = \frac{1}{\phi}\exp\{(\frac{y-\theta}{\phi}) - \exp[(y-\theta)/\phi]\}$$

   b. The log-normal distribution with density

   $$f_Y(y;\theta,\phi) = \frac{1}{y\sqrt{2\pi\phi}}\exp\{-\frac{1}{2\phi}(\log y - \theta)^2\}$$

3. For the classical linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}+\boldsymbol{\varepsilon}$, where $\mathbf{y}, \boldsymbol{\varepsilon}$ are n vectors, $\boldsymbol{\beta}$ has dimension p, $\mathbf{X}$ is $n \times p$, and the ε_i's are i.i.d. $N(0, \sigma^2)$, show that the information matrix of $\boldsymbol{\beta}$ is $\sigma^{-2}\mathbf{X}^T\mathbf{X}$.

4–10. In addition, do the following problems from Agresti: 3.25, 3.30, 4.5, 4.19, 4.22, 4.29, and 4.32.