Some Study Questions for Final Exam

1. The following table presents the results of a study examining student attitudes concerning traits they find desirable in mothers. Results are presented separately for male and female student respondents.

	Sex of Respondent		
Being a College Graduate	Male	Female	Total
Mentioned	26	16	42
Not Mentioned	343	407	750
Total	369	423	792

   Test the hypothesis that the gender of a respondent is independent of whether or not the respondent believes a college education is important in a mother. Use a 10% significance level.

Answer:

This hypothesis may be tested using Pearson's chi-square test of independence. Under the null hypothesis the statistic

$$X^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(n_{ij} - e_{ij})}{e_{ij}}$$

is approximately distributed as χ^2_1, where $e_{ij} = n_{i\cdot}n_{j\cdot}/n_{\cdot\cdot}$. The following table gives the expected cell counts (the e_{ij}'s):

	Sex of Respondent		
Being a College Graduate	Male	Female	Total
Mentioned	19.57	22.43	42
Not Mentioned	349.43	400.57	750
Total	369	423	792

$$\begin{aligned} X^2 &= \frac{(26-19.57)^2}{19.57} + \frac{(16-22.43)^2}{22.43} + \frac{(343-349.43)^2}{349.43} + \frac{(407-400.57)^2}{400.57} \\ &= 2.11 + 1.84 + 0.1183 + 0.1032 = 4.17 \end{aligned}$$

Our $p-$value is

$$p = \Pr[X^2 \geq 4.17]$$

so $0.05 > p > 0.025$ and we reject the null hypothesis.

2. The following table relates survival of infants to the amount of prenatal care received for 715 mothers:

Amount of Prenatal Care	Infants' Survival Died	Survived	Total
Less	20	373	393
More	6	316	322
Total	26	689	715

Test the hypothesis that infant survival is independent of the amount of prenatal care received by the mother. Use a 1% significance level.

Answer:

This hypothesis may be tested using Pearson's chi-square test of independence. Under the null hypothesis the statistic

$$X^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(n_{ij}-e_{ij})}{e_{ij}}$$

is approximately distributed as χ_1^2, where $e_{ij} = n_{i\cdot}n_{\cdot j}/n_{\cdot\cdot}$. The following table gives the expected cell counts (the e_{ij}'s):

Amount of Prenatal Care	Infants' Survival Died	Survived	Total
Less	14.29	378.71	393
More	11.71	310.29	322
Total	26	689	715

$$\begin{aligned} X^2 &= \frac{(20-14.29)^2}{14.29} + \frac{(373-378.71)^2}{378.71} + \frac{(6-11.71)^2}{11.71} + \frac{(316-310.29)^2}{310.29} \\ &= 2.28 + 0.08609 + 2.78 + 0.1051 = 5.25 \end{aligned}$$

Our $p-$valueis

$$p = \Pr[X^2 \geq 5.25]$$

so $0.025 > p > 0.01$ and we do not reject the null hypothesis at $\alpha = 0.01$.

3. In a study of uni-polar depression in married couples, 25 couples with a history of uni-polar depression in both partners were studied. Each individual in the study was examined by a psychiatrist and classified into one of the following four categories: mild, moderate, severe, or profound depression. The data are as follows:

	Level of Depression	
Couple No.	Husband	Wife
1	Mild	Moderate
2	Mild	Severe
3	Mild	Mild
4	Moderate	Mild
5	Moderate	Profound
6	Moderate	Moderate
7	Moderate	Mild
8	Mild	Profound
9	Mild	Severe
10	Severe	Severe
11	Severe	Profound
12	Severe	Mild
13	Severe	Severe
14	Moderate	Severe
15	Moderate	Moderate
16	Moderate	Profound
17	Moderate	Mild
18	Severe	Mild
19	Profound	Severe
20	Moderate	Severe
21	Mild	Mild
22	Mild	Moderate
23	Moderate	Mild
24	Severe	Profound
25	Mild	Severe

Suppose that we decide to simplify the diagnosis categories so that "Severe" and "Profound" are categorized as "Depressed" and "Mild" and "Moderate" are categorized as "Not Depressed".

a. Based on this simplified categorization, what is the appropriate statistical test to test the hypothesis that the probability of being "Depressed" is the same for husbands as it is for wives?

Answer:
McNemar's test.

b. Construct a contingency table appropriate for conducting McNemar's test on these data..

Answer:

		Diagnosis for Wife	
Diagnosis for Husband		Depressed	Not Depressed
	Depressed	5	2
	Not Depressed	8	10

c. Test the null hypothesis that the probability of depression among such couples is the same for husband and wife usingMcNemar's test. Use a 5% significance level.

Answer:

$$X^2 = \frac{(|O_{12} - O_{21}| - 1)^2}{O_{12} + O_{21}} = \frac{(|2 - 8| - 1)^2}{2 + 8} = 2.5$$

The critical value for an $\alpha = .05$-level test is $\chi^2_{1-.05}(1) = 3.84$, so since $X^2 = 2.5 < 3.84$ we fail to reject the hypothesis of equal probability of depression for husbands and wives from this population. The p-value for this test is

$$p = P(\chi^2(1) > 2.5) = 1 - P(\chi^2(1) \leq 2.5) = 1 - .8862 = .1138 \quad \text{(from Minitab)}$$

d. Is there any reason to be concerned over the use of McNemar's test in this problem? Why or why not, and if there is reason to be concerned, suggest an alternative test.

Answer:
The rule of thumb for using the approximate version of McNemar's test (the one used above) is that the total number of discordant pairs, $O_{12} + O_{21}$, be greater than or equal to 20. In this example, $O_{12} + O_{21} = 10$, so the sample size is not large enough to be confident that the p-value associated with McNemar's test will be accurate. The exact version of McNemar's test should be used here instead. This can be implemented in SAS and other statistical computer programs.

4. In a study of depression and electro-convulsive therapy (ECT), 65 patients with uni-polar depression that was described as either "Severe" or "Profound" prior to treatment were given ECT. It was found that 14 patients received the same assessment (severe or profound) before and after ECT, 28 received a milder assessment after ECT, and 23 patients received more severe assessment following ECT.

   a. Use McNemar's at a significance level of 10% to test the null hypothesis that ECT has no effect on the severity of depression among patients of the type studied here.

Answer:

$$X^2 = \frac{(|O_{12} - O_{21}| - 1)^2}{O_{12} + O_{21}} = \frac{(|28 - 23| - 1)^2}{28 + 23} = 0.706$$

The critical value for an $\alpha = .05$-level test is $\chi^2_{1-.10}(1) = 2.706$, so since $X^2 = 0.706 < 2.706$ we fail to reject the hypothesis that ECT has no effect on the severity of depression among patients from this population. The p-value for this test is

$$p = P(\chi^2(1) > .706) = 1 - P(\chi^2(1) \leq .706) = 1 - .5992 = .4008 \quad \text{(from Minitab)}$$

   b. Estimate the ratio of the odds of being profoundly depressed before ECT over the odds of being profoundly depressed after ECT, and place a 90% confidence interval around this quantity.

Answer:
$\hat{OR} = O_{12}/O_{21} = 28/23 = 1.22$. The odds of being profoundly depressed are 1.22 times higher (22% higher) before ECT than after. The standard error of $\ln(\hat{OR})$ is

$$\text{s.e.}\{\ln(\hat{OR})\} = \sqrt{\frac{O_{12} + O_{21}}{O_{12}O_{21}}} = \sqrt{\frac{28 + 23}{(28)(23)}} = .281$$

so an approximate 90% CI for $\ln(OR)$ is

$$\ln(\hat{OR}) \pm z_{1-.10/2}\text{s.e.}\{\ln(\hat{OR})\} = .1967 \pm 1.645(.281) = (-.266, .660)$$

Therefore, an approximate 90% CI for OR is

$$(e^L, e^U) = (e^{-.266}, e^{.660}) = (.766, 1.93)$$

5. To study the role of genetics in heart disease (HD) the medical histories of 100 randomly selected deceased men were examined. In addition, the medical records of the fathers of these men were examined. From these records the following information was gathered:

	Father		
Son	HD	No HD	Total
HD	15	12	27
No HD	18	55	73
Total	33	67	100

Test the hypothesis that the probability of HD is the same among fathers and sons at the 10% significance level.

Answer:
Here $O_{12} = 12$, $O_{21} = 18$. Since $O_{12} + O_{21} = 30$, McNemar's test may be used. The test statistic is

$$X^2 = \frac{(|O_{12} - O_{21}| - 1)^2}{O_{12} + O_{21}} = \frac{(|12 - 18| - 1)^2}{30} = .833$$

The $\alpha = .10$ critical value is $\chi^2_{1-.10}(1) = 2.706$. Since $.833 < 2.706$, we fail to reject H_0. There is insufficient evidence here to conclude that the probability of HD is the same for fathers as for their sons. The approximate p-value is

$$P(\chi^2(1) > .833) = 1 - P(\chi^2(1) \leq .833) = 1 - .6387 = .3613.$$

6. In a study of respiratory illness a sample of 187 children whose mothers smoked were followed over time as they aged from 7 to 10 years of age. The following table summarizes their respiratory illness status (wheeze, no wheeze) at ages 7 and 10:

	Age 10		
Age 7	Wheeze	No Wheeze	Total
Wheeze	13	18	31
No Wheeze	13	143	156
Total	26	161	187

a. Test the hypothesis that the probability of wheezing is the same at ages 7 and 10 at the 5% significance level. Perform the test using an approximate method.

Answer:
Again, this is a problem for which McNemar's test is appropriate. The test statistic is

$$X^2 = \frac{(|O_{12} - O_{21}| - 1)^2}{O_{12} + O_{21}} = \frac{(|18 - 13| - 1)^2}{31} = .516$$

The $\alpha = .05$ critical value is $\chi^2_{1-.05}(1) = 3.84$. Since $.516 < 3.84$, we fail to reject H_0. There is insufficient evidence here to conclude that the probability of wheezing is the same at age 7 as it is at age 10. The approximate p-value is

$$P(\chi^2(1) > .516) = 1 - P(\chi^2(1) \leq .516) = 1 - .5275 = .4725.$$

b. Estimate the odds ratio comparing the odds of wheezing at age 7 to the odds of wheezing at age 10 and place a 95% confidence interval around this quantity.

Answer:
$\hat{OR} = O_{12}/O_{21} = 18/13 = 1.38$. The odds of wheezing are 1.38 times higher (38% higher) at age 7 than at age 10. The standard error of $\ln(\hat{OR})$ is

$$\text{s.e.}\{\ln(\hat{OR})\} = \sqrt{\frac{O_{12} + O_{21}}{O_{12}O_{21}}} = \sqrt{\frac{18 + 13}{(18)(13)}} = .3640$$

so an approximate 95% CI for $\ln(OR)$ is

$$\ln(\hat{OR}) \pm z_{1-.05/2}\text{s.e.}\{\ln(\hat{OR})\} = .3254 \pm 1.96(.3640) = (-.388, 1.039)$$

Therefore, an approximate 95% CI for OR is

$$(e^L, e^U) = (e^{-.388}, e^{1.039}) = (.678, 2.83)$$

7. Suppose that the probability that a child born to a woman 40 years of age or older has Down's Syndrome is 0.0086. Of 1000 women aged 40 years and above, 21 gave birth to children with Down's Syndrome. Is this unusually high?

Answer:
Let p = Pr[woman gives birth to child with Down's Syndrome]. We want to test $\text{H}_0 : p = 0.0086$ versus $\text{H}_A : p > 0.0086$.

Exact answer:
Let X = the number of women who give birth to a Down's Syndrome child, then under H_0, $X \sim B(500, 0.0086)$. The $p-$ value is

$$P(X \geq 21) = 1 - P(X \leq 20) = 1 - .999772 = .000228$$

Approximate answer:
Alternatively, we can use a z test. In this case, our test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{.021 - .0086}{\sqrt{.0086(1 - .0086)/1000}} = 4.247$$

The approximate p value is

$$P(Z > 4.247) = .00001$$

Since the p-value is less than $\alpha = .05$ we reject H_0 and conclude that the probability in the population from which this sample is drawn has a population proportion greater than .0086. So, yes, this is an unusually high observed number of cases of Down's syndrome. Note that the approximate z test used here is appropriate since the expected number of cases under H_0 is $np_0 = 1000(.0086) = 8.6 \geq 5$.

8. As part of a survey of public opinion concerning national politics, in October, 1993, each member of a random sample of 550 Americans was asked whether or not he/she approved of the job that Bill Clinton is doing as President. Of the 550 respondents, 210 people said that they approved of Bill Clinton's performance as president. Find an approximate 90% confidence interval for the true percentage of the U.S. population that approves of Bill Clinton's performance.

Answer:
If Y = the number of respondents who approve then $Y \sim B(550, \pi)$. An approximate $100(1-\alpha)\%$ confidence interval for p is given by

$$\left(\hat{p} - z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

In this case we have

$$\left((210/550) - z_{1-0.1/2}\sqrt{\frac{(210/550)(1-210/550)}{550}}, (210/550) + z_{1-0.1/2}\sqrt{\frac{(210/550)(1-210/550)}{550}}\right)$$

or, equivalently,

$$\left((210/550) - 1.645\sqrt{\frac{(210/550)(1-210/550)}{550}}, (210/550) + 1.645\sqrt{\frac{(210/550)(1-210/550)}{550}}\right)$$

or, equivalently,

$$(0.3532, 0.4104)$$

9. Refer to the previous problem. Suppose that in October, 1994 each member of another random sample of 650 Americans was asked whether or not he/she approved of Bill Clinton's performance as President. In this sample 302 people expressed approval of Clinton's performance. Test whether or not Clinton's approval ratings changed between 1993 and 1994. Use a 5% significance level.

Answer:
Let p_1 = the proportion of Americans who approved of Clinton in 1993 and let p_2 = the proportion of Americans who approved of Clinton in 1994. Test $\text{H}_0 : p_1 - p_2 = 0$ vs. $\text{H}_A : p_1 \neq p_2$ at $\alpha = 0.05$.

This can be done with either a z test or, equivalently, with a Pearson chi-square test. Here, I will use the z test. The test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})[1/n_1 + 1/n_2]}}$$

where $\hat{p}_1 = 210/550 = .3818$, $\hat{p}_2 = 302/650 = .4646$ and $\hat{p} = (210+302)/(550+650) = .4267$. So,

$$z = \frac{.3818 - .4646}{\sqrt{.4267(1-.4267)[1/550 + 1/650]}} = -2.8894$$

The p-value is

$$p = 2P(Z > |z|) = 2P(Z > 2.8894) = 2(.00193) = .0039$$

Therefore, we reject H_0 and conclude that Clinton's approval rating changed (increased in this case) between 1993 and 1994. Here the sample size was large enough to justify the z test since $n_1\hat{p}_1 = 210$, $n_1(1-\hat{p}_1) = 340$, $n_2\hat{p}_2 = 302$, and $n_2(1-\hat{p}_2) = 348$ are all ≥ 5.