Example:

   An experiment is designed to study pigment dispersion in paint. Four different methods of mixing a particular pigment are studied. The procedure consists of preparing a gallon can of paint using one of the four mixing methods and then applying each of the three one-third portions of that gallon to a wood panel using one of three application methods (brushing, rolling, and spraying). Because of the amount of time required to do the painting, the experiment was conducted over three consecutive days, with four gallons of paint (one with each of the four mixing methods) applied each day. The data are given below and are analyzed in pigment.sas which you should copy from the course website and run in SAS.

Day	Application Method	Mixing Method 1	2	3	4
	brushing	64.5	66.3	74.1	66.5
1	rolling	68.3	69.5	73.8	70.0
	spraying	70.3	73.1	78.0	72.3
	brushing	65.2	65.0	73.8	64.8
2	rolling	69.2	70.3	74.5	68.3
	spraying	71.2	72.8	79.1	71.5
	brushing	66.2	66.5	72.3	67.7
3	rolling	69.0	69.0	75.4	68.6
	spraying	70.8	74.2	80.1	72.4

   This experiment has the same design as the alfalfa example discussed in class. The appropriate model for this design is

$$y_{ijk} = \mu + \alpha_i + \tau_j + e_{ij} + \beta_k + (\alpha\beta)_{ik} + \varepsilon_{ijk}$$

where y_{ijk} is the percentage reflectance measured on the wood panel that was painted with application method k, on day j and with paint mixed with mixing method i. μ is a grand mean, α_i is a (fixed) effect for mixing method i (the whole plot factor), τ_j is a (random) effect for day j, e_{ij} is the whole plot error term, β_k is a (fixed) effect for the kth application method (the split-plot factor), $(\alpha\beta)_{ik}$ is a (fixed) effect for the ith mixing method when it is combined with application method k (it is the interaction effect), and ε_{ijk} is the split-plot error term.

Examine pigment.sas and its associated output. Here we illustrate how to fit a split plot model in PROC MIXED.

- In the first call to PROC MIXED we fit the model described above to these data. One feature of this model is that it does not account for the possibility of a DAY*APPMETH interaction. That is, it implicitly assumes that differences in application method (the split-plot factor) do not change across days (the whole plot blocking factor). This assumption is very often reasonable a priori and I would say that it seems reasonable in the present context. If you don't think this assumption is reasonable in any given application, then the analysis could be done based upon the model that includes this interaction, but we won't do that here.

- In pigment.sas I have used method=type3 to fit the model. The results for method=reml are essentially the same except for the method with which confidence intervals for variance components are computed. Although I think the method used under type=REML is better, I wanted to use type=type3 because it generates the ANOVA table. If you look at that ANOVA you will see that SAS prints out the expected mean squares for each source of variability and then also identifies which MS is the appropriate error term to test that source of variablity.

  - For instance, for mixmeth (the whole plot factor) and day (the blocking factor at the whole plot level), the appropriate error term is identified as MS(mixmeth*day), which is $MS_{E(Wholeplot)}$. For appmeth (the split plot factor) the appropriate error term is correctly identified as MS(residual), which is $MS_{E(splitplot)}$.

- Also produced in this analysis are the model-based estimates of the marginal means for mixmeth and appmeth, and of the joint means. Recall that the estimators of these quantities are all simple sample means. However, the standard errors for the marginal means for factor B (appmeth) and for the joint means are functions of both of the MS_Es in the model. Therefore Satterthwaite's method is needed to compute approximate t confidence intervals for the marginal means for appmeth and the joint means. Satterthwaite's method is requested with the ddfm=satterth option on the MODEL statement. Evidence that it has been used is provided by the degrees of freedom that SAS gives in the table of Least Squares Means on p.3 of pigment.pdf. The fractional degrees of freedom are the ones estimated using Satterthwaite's formula.

- For illustration, I have also asked for some estimates and tests of contrasts involving the joint means. In particular, suppose we'd like to compare brushing and rolling when paint is mixed with method 1. This contrast is $\gamma = \bar{\mu}_{1\cdot1} - \bar{\mu}_{1\cdot2}$ ($k = 1$ refers to brushing, $k = 2$ to rolling). This contrast is a comparison of joint means across the levels of factor B within a level of A. That is, it does not make a comparison across the levels of factor A, so for such a contrast, Satterthwaite's formula is not needed. This can be seen by the fact that neither our estimate of this contrast nor our test of it ($F_{1,16} = 27.88$, $p < .0001$) has fractional degrees of freedom (see p.3). To get the coefficients for this contrast, recall that SAS uses

the effects model parameterization, and we have to translate our coefficients on the $\bar{\mu}_{i\cdot k}$ into coefficients on the μ, α_i's, β_k's, $(\alpha\beta)_{ik}$'s, etc. We do this as follows. According to our model,

$$y_{ijk} = \mu + \alpha_i + \tau_j + e_{ij} + \beta_k + (\alpha\beta)_{ik} + \varepsilon_{ijk}$$

so the mean $\mu_{ijk} = \mathrm{E}(y_{ijk})$ is given by

$$\mu_{ijk} = \mathrm{E}(\mu + \alpha_i + \tau_j + e_{ij} + \beta_k + (\alpha\beta)_{ik} + \varepsilon_{ijk}) = \mu + \alpha_i + \tau_j + \beta_k + (\alpha\beta)_{ik}$$

and therefore,

$$\bar{\mu}_{i\cdot k} = \frac{1}{n}\sum_{j=1}^{n}\mu_{ijk} = \frac{1}{n}\sum_{j=1}^{n}(\mu + \alpha_i + \tau_j + \beta_k + (\alpha\beta)_{ik})$$
$$= \mu + \alpha_i + \bar{\tau}. + \beta_k + (\alpha\beta)_{ik}$$

Therefore our contrast $\bar{\mu}_{1\cdot 1} - \bar{\mu}_{1\cdot 2}$ can be written in terms of the effects parameters as

$$\bar{\mu}_{1\cdot 1} - \bar{\mu}_{1\cdot 2} = \mu + \alpha_1 + \bar{\tau}. + \beta_1 + (\alpha\beta)_{11}$$
$$- \mu - \alpha_1 - \bar{\tau}. - \beta_2 - (\alpha\beta)_{12}$$
$$= \beta_1 - \beta_2 + (\alpha\beta)_{11} - (\alpha\beta)_{12}$$

This is specified on the CONTRAST statement as

CONTRAST 'brsh vs roll, meth 1' APPMETH 1 -1 0 MIXMETH*APPMETH 1 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 ;

- If instead we wanted to test a joint mean contrast across levels of factor A, mixing method, such as $\gamma_2 = \bar{\mu}_{1\cdot 1} - \bar{\mu}_{3\cdot 3}$ (brushing with mixing method 1 versus spraying with mixing method 3), Satterthwaite's approximation would be needed. We can see this from the fractional degrees (21.2) given for this contrast on p.3 of pigment.pdf.

    – Note that PROC MIXED takes care of all of this for us. It knows when Satterthwaite's formula is needed and when it is not, and produces correct estimates, standard errors, confidence intervals and tests automatically.

- Note that in the analyses described above, we have treated DAY, the whole plot blocking factor, as having fixed rather than random levels. This is not really the most appropriate modelling assumption here. It makes better sense to think of the particular days involved in this experiment as representative of the population of all days on which this painting could have been done. That is, we ought to assume that $\tau_1, \ldots, \tau_n \overset{iid}{\sim} N(0, \sigma_\tau^2)$ are random effects rather than fixed. Treating DAY effects as random can be accomplished fairly easily in PROC MIXED by simply moving DAY from the MODEL statement to the RANDOM statement. This has a very small effect on the analysis: it does not change any of the tests on the fixed

effects or contrasts, but it does change the standard errors for the marginal and joint means slightly. The results from this final call to PROC MIXED (pp.5–8) where DAYs are treated as random, are really the most appropriate analysis of this example.

- Finally, after talking about the analysis methods, we should actually look at the analysis and draw some conclusions. On p.7 we see that the mixmeth*appmeth interaction is nearly significant ($F_{6,16} = 2.49$, $p = .0678$). However, an examination of the profile plots on pp. 9–10 reveals that whatever interaction there may be, it is an orderly one; across all four mixing methods, spraying has the highest mean followed by rolling and then brushing. Similarly, over all three application methods, the mixing methods have roughly the same relationship (mixing method 3 has a much larger mean than the others, which are all quite similar). Therefore, it is meaningful to test main effects of factors A and B and to make inference on the marginal means for these factors. The main effect tests reveal that the mean response differs across both mixing methods and application methods ($p < .0001$ in each case). In addition, pairwise comparisons with the best level of each factor (mixing method 3, and the "spraying" application method) show that these factor levels are significantly better ($p < .0001$ in each case) than each other level of their respective factors.

Name:

Exercise:

Steel is normalized by heating above the critical temperature, soaking, and then air cooling. This process increases the strength of the steel, refines the grain, and homogenizes the structure. An experiment is performed to determine the effect of temperature and heat treatment time on the strength of normalized steel. Two temperatures and three times are selected. The experiment is performed by heating the oven to a randomly selected temperature and inserting the three specimens. After 10 minutes one specimen (selected at random) is removed, after 20 minutes a second specimen is removed, and after 30 minutes the remaining specimen is removed. Then the temperature is changed to the other level and the process repeated. Four shifts are required to collect the data, which follow:

		Temperature(°F)	
Shift	Time(min.)	1500	1600
	10	63	89
1	20	54	91
	30	61	62
	10	50	80
2	20	52	72
	30	59	69
	10	48	73
3	20	74	81
	30	71	69
	10	54	88
4	20	48	92
	30	59	64

a. Identify the design used here.

b. What is the appropriate model for analyzing this design?

c. Write a SAS program to obtain the correct ANOVA Table for this design. Also include in your SAS program a contrast for the linear effect of time (coefficients are -1 0 1) and a contrast to compare the two temperatures. Make sure that your SAS program estimates means for time, temperature and time*temperature. The data for this problem are in the public data directory in the file steel.dat.

d. Report F tests and p-values for main effects and interactions based on your fitted model.

Please hand in pp.5–6, including your answers. Remember to write your name at the top. You may keep pages 1–4 for your notes.