Estimating optimum sampling size to determine weighted core specific gravity of planted loblolly pine

Lewis Jordan, Laurence R. Schimleck, Alexander Clark III, Daniel B. Hall, and Richard F. Daniels

Abstract: Data from a variability study of loblolly pine (*Pinus taeda* L.) based on weighted core specific gravity (WCSG) were examined to show how costs and variance estimates are used in designing efficient sampling strategies. Increment cores for the determination of WCSG were taken from 3957 trees across six distinct physiographic regions in the southeastern United States. More variability was found to exist among stands than within stands. This indicates that reducing the variation of the mean of WCSG can be accomplished by sampling more stands and fewer trees in the region of interest. The number of stands and trees to sample is dictated by the maximum allowable cost and the precision required of the sample mean, and formulas are given for such calculations. The estimate of among-stand variability was found to be similar among the regions of interest, whereas larger within-stand variation was found to exist in the South Atlantic and Hilly regions. The standard error of the mean was found to increase with an increase in the age at which the stand was sampled. When sampling across multiple stands (at any age), little if any gain in the precision of the standard error of the mean is gained by sampling more than 15 trees. In the general case where one is interested only in the value of WCSG in one stand and precision or cost–time factors are not of consideration, it would suffice to sample between 45 and 55 trees at any age.

Résumé : Des données provenant d'une étude de variabilité basée sur le poids spécifique pondéré de carottes de pin à encens (*Pinus taeda* L.) ont été examinées pour illustrer comment l'estimation des coûts et de la variance est utilisée pour élaborer des stratégies efficaces d'échantillonnage. Les carottes utilisées pour déterminer le poids spécifique pondéré ont été prélevées sur 3957 arbres répartis dans six régions physiographiques distinctes du sud-est des États-Unis d'Amérique. Une plus grande variabilité a été observée entre les peuplements que dans les peuplements. Cela signifie qu'il est possible de réduire la variation de la moyenne du poids spécifique pondéré en échantillonnant plus de peuplements et moins d'arbres dans la région visée. Le nombre de peuplements et d'arbres à échantillonner est dicté par le coût maximal admissible et la précision de la moyenne de l'échantillon qui est requise; des formules servant à effectuer ces calculs sont fournies. L'estimation de la variabilité entre les peuplements était semblable d'une région à l'autre tandis qu'une plus forte variabilité a été observée dans les peuplements des régions de South Atlantic et de Hilly. L'erreur type de la moyenne augmente avec l'âge auquel le peuplement a été échantillonné. Lorsqu'on échantillonne plusieurs peuplements d'âges différents, on obtient peu sinon aucun gain de précision de l'erreur type de la moyenne en échantillonnant plus de 15 arbres. En général, lorsqu'on s'intéresse seulement à la valeur du poids spécifique pondéré dans un peuplement et que la précision ou les facteurs de coût ou de temps ne sont pas importants, il serait suffisant d'échantillonner 45 à 55 arbres peu importe l'âge.

[Traduit par la Rédaction]

Introduction

The southeastern United States produces 58% of the marketed timber in the United States and 16% of all marketed timber in the world (Wear and Greis 2002). Loblolly pine (*Pinus taeda* L.) is the most important commercial softwood species in this region and accounts for the majority of timber utilized. Loblolly pine is a desired wood resource for the structural lumber, plywood, and pulping industries. Megraw (1985) states: "Of all the parameters practical to measure, specific gravity is recognized as the most useful index to predict the physical behavior of wood." Because it is closely related to the strength and stiffness of wood, specific gravity (SG) is used as a primary factor in the segregation of high-strength lumber, poles, and pilings and largely determines pulp yield (Koch 1972; Zobel and Van Buijtenen 1989). Therefore, SG is important in determining the value and utility of wood, surpassing the importance of many other wood properties.

Variations in SG of any tree species can be attributed to

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L. Jordan,[1] L.R. Schimleck, and R.F. Daniels. Warnell School of Forestry and Natural Resources, University of Georgia, Athens, GA 30602, USA.
A. Clark, III. USDA Forest Service, Southern Research Station, Athens, GA 30602, USA.
D.B. Hall. Department of Statistics, University of Georgia, Athens, GA 30602, USA.

[1]Corresponding author (e-mail: lcj6444@warnell.uga.edu).

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variation within a tree, among trees in a particular stand, among different growing sites, and among different silvicultural regimes (Addis et al. 1995). Within an individual tree, SG varies across each annual ring, among adjacent rings, with radial position in the stem, and with height aboveground (Koch 1972). SG generally decreases with increasing height and increases with an increase in ring number from pith. SG has been found to increase sharply with radial distance from pith until an annual age ranging from 5 to 10 years is reached, at which point SG continues to increase slowly until ring 30 (Koch 1972).

In forestry sampling applications, it is customary to measure all trees in a plot to determine the variable of interest. However, when sampling for properties that are expensive and time consuming to measure, it is uneconomical to measure all trees. It is then essential to develop a sampling design that will minimize the cost of obtaining the sample estimate if the desired degree of precision is fixed, or conversely, to maximize the precision of the estimate obtained from a given expenditure of personnel, time, and equipment (Marcuse 1949). One way to minimize the cost or time variance product is to subsample. In two-stage sampling, the population is subdivided into n primary sampling units (stands, plots, and trees), from which a sample of m secondary sampling units (plots within stands, trees within plots, and disks within trees) are chosen, where n and m represent the sample sizes, respectively.

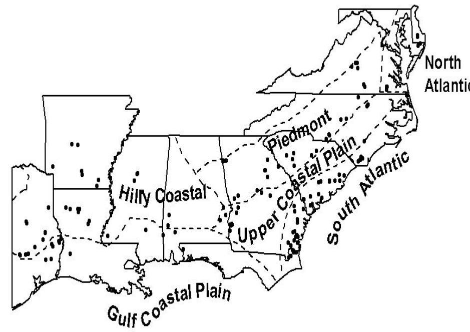
Within the southeastern United States, traditionally 30 trees per stand have been selected for weighted core SG (WCSG) analysis (Clark and Ike 1970). This sample size has also historically been employed in New Zealand for radiata pine (*Pinus radiata* D. Don) (Raymond 2006). In a study of spruce pine (*Pinus glabra* Walt.) from 35 locations, Clark and Ike (1970), using a nested analysis of variance, found that the optimum number of trees to sample for WCSG was between 10 and 20 trees, a reduction from the 33 trees that were originally sampled in their study. Clark and Ike emphasize that sampling more locations is more important in reducing the mean variance than is the number of trees sampled per location. Recently, Raymond (2006) suggested that the number of sample cores selected for SG analysis in radiata pine can be reduced from 30 to 10. Raymond (2006) examined the effect of reducing the sample size on the variability of wood density for each of 36 sites, on the basis of having randomly sampled 5, 10, 15, 20, or all 30 trees from each site. These results indicated little change in the site mean when sampling was increased to more than 10 trees per site.

The objective of this paper is to estimate the precision and cost of determining WCSG of loblolly pine in the southeastern United States and to evaluate the precision and cost differences under differing sampling intensities using a two-stage sampling design.

Materials and methods

One-hundred and forty-seven planted loblolly pine stands were selected on private industry landholdings in six distinct physiographic regions (Fig. 1). Stands selected for sampling were conventionally managed and did not receive any intensive silvicultural practices, such as chemical competition control or fertilization, except phosphorous on phosphorous-deficient sites. At the time of sampling, stocking ranged from 494 to 1482 trees/ha, but stocking at time of planting was generally unknown. A sample of 15–30 trees was randomly selected in proportion to the diameter distribution of the trees in the stand. Increment cores, 12 mm in diameter, were extracted bark to bark through the pith from those trees selected for sampling at 1.37 m from the ground.

Fig. 1. Location of loblolly pine stands sampled to determine weighted core specific gravity analysis.



Immediately after removal from the tree, the increment cores were stored in ice and subsequently shipped in a green condition to the USDA Forest Service Southern Research Station in Athens, Georgia, for analysis. The cores were divided into two radii then dried at 50 °C, and one radius was glued to core holders and sawn into 1.6 mm thick radial strips. The radial strips were conditioned to 8% equilibrium moisture content and then read on a Quintek Measurement Systems^{TM} scanning X-ray densitometer (Quintek Measurement Systems, Knoxville, Tenn.) at a resolution of 0.06 mm to determine radial growth of earlywood and latewood and SG of earlywood and latewood of each annual ring. The SG data were collected on the longitudinal surface. SG values were based on a green volume and oven dry mass basis, with a value of 0.48 used to distinguish between earlywood and latewood SG. Mean annual ring SG was weighted by ring basal area to obtain WCSG. Regional attributes are summarized in Table 1.

Statistical analysis

The statistical analysis performed in this paper was based on a two-stage design. It was assumed that n, the number of primary sampling units (stands) selected for sampling in each region, was from an infinite population. It was also assumed that m, the number of secondary sampling units (trees) selected for sampling in each stand, was from an infinite population. This is justified as the ratio of n/N, where N is the total stands in the region of interest, and m/M, where M is the total number of trees per stand, is negligible as is the case for the WCSG data, where the number of stands and trees sampled is small in comparison to the total number of stands and trees in the regions. A derivation based on the infinite population theory is often employed (Marcuse 1949; Brooks 1955; Zarnoch et al. 1993, 2004) and is more familiar to those acquainted with the analysis of variance (ANOVA). It is also assumed that any optimum solution found is relative only to homogeneous stands of the nature described in the Materials and methods section.

Table 1. Tree and stand characteristics selected for sampling by physiographic region.

Region	No. of stands sampled	No. of trees sampled	DBH (cm)	Total height (m)	Age (years)	Site index (m)
South Atlantic	43	1048	24.8 (6.0)	21.0 (3.4)	23.4 (4.1)	22.6 (2.4)
North Atlantic	7	207	21.5 (5.4)	17.4 (2.6)	22.4 (1.9)	19.4 (2.18)
Upper Coastal	17	495	22.4 (5.5)	18.4 (3.2)	22.9 (1.4)	20.0 (2.61)
Piedmont	30	813	23.1 (5.3)	17.9 (2.8)	24.1 (4.5)	18.9 (2.0)
Gulf	17	492	20.5 (4.3)	18.9 (3.1)	23.5 (3.6)	20.5 (2.5)
Hilly	33	902	23.4 (5.7)	19.1 (3.1)	23.9 (4.3)	20.4 (2.6)

Note: Values for diameter at breast height (DBH), height, age, and site index are means with SDs given in parentheses, which were calculated by pooling the data in each region.

Table 2. ANOVA (eq. 1) results including the number of stands sampled (n_i), the mean number of cores per stand ($\bar{m}_{ij}$), along with corresponding estimated variance components, and the estimated value of WCSG ($\hat{\mu}$) by region.

Region	n_i	$\bar{m}_{ij}$	$\hat{\sigma}^2_{e_i}$	$\hat{\sigma}^2_{S_i}$	$\hat{\mu}$
South Atlantic	43	24.37	0.001 412	0.000 393	0.5276
North Atlantic	7	29.57	0.001 088	0.000 302	0.4817
Upper Coastal	17	29.11	0.001 118	0.000 509	0.4927
Piedmont	30	27.10	0.001 097	0.000 596	0.4860
Gulf	17	28.94	0.001 136	0.000 311	0.5008
Hilly	33	27.33	0.001 301	0.000 309	0.4797

Let y_{ijk} equal the WCSG value of the kth tree in the jth stand of the ith region (i = 1, 2, ..., R; j = 1, 2, ..., n_i; k = 1, 2, ..., m_{ij}); R equal the number of regions (6); n_i equal the total number of stands selected for sampling in the ith region of interest; and m_{ij} equal the number of trees selected for sampling in the jth stand of the ith region. An appropriate linear model can be written as

$$[1] \qquad y_{ijk} = \mu_i + S_{j(i)} + e_{ijk}$$

where μ_i equals the mean value of WCSG in the ith region; $S_{j(i)}$ equals the effect of the jth randomly selected stand in the ith region, with $S_{j(i)} \sim \mathrm{N}(0,\sigma^2_{S_i})$; e_{ijk} is residual error, with $e_{ijk} \sim \mathrm{N}(0,\sigma^2_{e_i})$, where $\sigma^2_{S_i}$ and $\sigma^2_{e_i}$ represent among- and within-stand components of variance, respectively, which we index by i, to denote that these parameters are allowed to differ across regions.

An estimator of the mean for the ith region is

$$[2] \qquad \hat{\mu}_i = \frac{1}{n_i m_{ij}} \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} y_{ijk}$$

with estimated variance

$$[3] \qquad \hat{\sigma}^2(\hat{\mu}_i) = \frac{\sigma^2_{S_i}}{n_i} + \frac{\sigma^2_{e_i}}{n_i m_{ij}}$$

When the number of trees sampled per stand is not constant, an estimate of the mean number of trees per stand m^*_{ij} is used for m_{ij} and is defined as

$$[4] \qquad m^*_{ij} = \frac{1}{n_i - 1} \left(\sum_{j=1}^{n_i} m_{ij} - \frac{\sum_{j=1}^{n_i} m^2_{ij}}{\sum_{j=1}^{n_i} m_{ij}} \right)$$

As stated by Zarnoch et al. (2004), eq. 3 gives the sample variance under any alternative combination of stands and trees per stand. Thus, the standard error of $\hat{\mu}_i$ for any n_i and m_{ij} combination can be found as $\sqrt{\hat{\sigma}^2(\hat{\mu}_i)}$ and confidence intervals at the 1– α level are

$$[5] \qquad \hat{\mu}_i \pm t_{(1-\alpha),(n_i-1)} \sqrt{\hat{\sigma}^2(\hat{\mu}_i)}$$

where $t_{(1-\alpha)(n_i-1)}$ is the two-tailed 1 – α value from the t distribution with (n_i – 1) degrees of freedom.

We can consider eq. 1 to be a fully saturated model, fitting separate variance components (VCs) $\sigma^2_{S_i}$ and $\sigma^2_{e_i}$ for each region. However, in some cases a more parsimonious model may be utilized, fitting a common σ^2_S and σ^2_e VC. Testing the equality of $\sigma^2_{S_i}$ is equivalent to fitting eq. 1 with σ^2_S or a pooled VC. This is a test of $\sigma^2_{S_1} = \sigma^2_{S_2} = \ldots = \sigma^2_{S_6}$ (six regions) and can be accomplished via a likelihood ratio test that is asymptotically distributed as χ^2_5. A similar test can be performed to determine if a common σ^2_e VC can be utilized in the model. Estimates of the VC were obtained using the SAS MIXED procedure (SAS Institute Inc. 2000–2004).

Optimum allocation

Optimum allocation can be expressed as a function of the variance of the sample mean and the total cost (or time) expenditure for determining it. Considering the case of two levels, let C (n, m) be the cost function and V (n, m) the variance function, where the variables n and m represent the sample sizes, respectively. The cost function is assumed to be an additive function of the costs at the two levels, that is, the cost of n primary units and nm secondary units, with the cost per primary and secondary sampling units being c_1 and c_2, respectively, yields

$$[6] \qquad C = c_1 n + c_2 nm$$

The primary unit cost ($c_1 n$) is proportional to the number of primary units (stands) in the sample and the secondary unit cost ($c_2 nm$) being proportional to the total number of secondary units (trees).

The goal of optimum allocation is to minimize C (n, m) given a prespecified fixed variance, V_f, or to minimize V (n, m) subject to the constraint of a total fixed cost, C_f. The optimum number of primary and secondary units for fixed variance and cost is given by Waters and Chester (1987) and Cohen (1998), respectively:

$$[7] \qquad \hat{n}_{\text{opt},v} = \frac{(\sigma_S^2 + \sigma_e^2/\hat{m}_{\text{opt}})}{V_f}, \hat{m}_{\text{opt}} = \sqrt{\frac{c_1}{c_2}}\frac{\sqrt{\sigma_e^2}}{\sqrt{\sigma_S^2}}$$

$$[8] \qquad \hat{n}_{\text{opt},c} = \frac{C_f}{(c_1 + c_2\hat{m}_{\text{opt}})}, \hat{m}_{\text{opt}} = \sqrt{\frac{c_1}{c_2}}\frac{\sqrt{\sigma_e^2}}{\sqrt{\sigma_S^2}}$$

As stated by Marcuse (1949), the optimum number of secondary units is independent of the degree of precision or the fixed cost under the infinite population assumption. This implies that, when planning an experiment, one needs only be concerned with the fixed cost or precision in selecting the number of primary sampling units. From this, an increase in funds is best allocated to sampling more primary units. The number of trees per stand to sample must be an integer, but the estimate of $\hat{m}_{\text{opt}}$ will usually be a fractional value. Following Cameron (1951), let the nearest integer below $\hat{m}_{\text{opt}}$ be m and let that above it be $m + 1$. From eqs. 3 and 8, the most information for least cost will be given using $m + 1$ trees when $(c_1/c_2)(\sigma_S^2/\sigma_e^2) > m(m+1)$; otherwise, select m trees.

Given values of σ_S^2 and σ_e^2, different combinations of n and m yield the same variance, signifying a trade-off between n and m in the attainment of equal variance (Waters and Chester 1987). An increase in either n or m reduces variance but increases sampling costs, whereas decreasing n or m increases variance and decreases sampling costs. Optimal allocation depends on (*i*) σ_S^2/σ_e^2, the relative variability between and within primary units; (*ii*) the ratio of c_1/c_2; and (*iii*) the desired precision V_f, for $\hat{\mu}$. The optimal sample design calls for a larger number of primary units when σ_S^2 is large and, a greater number of subsamples when σ_e^2 is large.

Results and discussion

The estimates of the VC as described above were obtained for the full model (eq. 1) (Table 2). The estimates of $\sigma_{S_i}^2$ indicate that, generally, the among-stand VCs are relatively homogeneous, with the possible exception of the Upper Coastal and Piedmont regions. We then tested the hypothesis $\sigma_{S_1}^2 = \sigma_{S_2}^2 = \ldots = \sigma_{S_6}^2$ or a common among-stand VC. The value of the test statistic, or the differences of twice the negative log-likelihoods (–2LL), between the full and reduced models was found to be $(-14911.7 + 14915.5) = 3.8 \sim \chi_5^2$. The resulting p value was 0.5786, indicating that the reduced model is preferred and that a common stand VC of σ_S^2 will be utilized. We then tested the hypothesis of $\sigma_{e_1}^2 = \sigma_{e_2}^2 = \ldots = \sigma_{e_6}^2$ or a common within stand VC. The –2LL was found to be $(-14890.4 + 14911.7) = 21.3 \sim \chi_5^2$, with a p value of 0.0007, indicating that separate estimates of $\sigma_{e_i}^2$ are required. From Table 2, it can be seen that the estimates of $\sigma_{e_i}^2$ appear to be similar in the North Atlantic, Piedmont, Upper Coastal, and Gulf (NPUG) regions compared with the South Atlantic and Hilly (SH) regions. From this, we then estimated separate VC for the NPUG and SH groups to test the hypothesis that $\sigma_{e_1}^2 = \sigma_{e_2}^2 = \sigma_{e_3}^2 = \sigma_{e_4}^2$ and $\sigma_{e_5}^2 = \sigma_{e_6}^2$. The resulting –2LL value was $(-14909.9 + 14911.7) = 1.8 \sim \chi_4^2$, with a p value of 0.7724, indicating that grouping the within stand VCs is appropriate. The estimates of the VCs for the final reduced model are $\hat{\sigma}_S^2$ = 0.000 418, $\hat{\sigma}_{e_{\text{NPUG}}}^2$ = 0.001 111, and $\hat{\sigma}_{e_{\text{SH}}}^2$ = 0.001 360.

Table 3. Estimated standard error associated with sample size options of n (stands) and m (trees).

n	m	Total no. of trees	NPUG	SH
10	5	50	0.008 00	0.008 31
	15	150	0.007 01	0.007 13
	30	300	0.006 75	0.006 81
20	5	100	0.005 66	0.005 87
	15	300	0.004 96	0.005 04
	30	600	0.004 77	0.004 81
40	5	200	0.004 00	0.004 15
	15	600	0.003 51	0.003 57
	30	1200	0.003 37	0.003 40
75	5	375	0.002 92	0.003 03
	15	1125	0.002 56	0.002 60
	30	2250	0.002 46	0.002 49
125	5	625	0.002 26	0.002 35
	15	1875	0.001 98	0.002 02
	30	3750	0.001 91	0.001 93

Note: NPUG, North Atlantic, Piedmont, Upper Coastal, and Gulf regions; SH, South Atlantic and Hilly regions. $\hat{\sigma}_S^2$ = 0.000 418, $\hat{\sigma}_{e_{\text{NPUG}}}^2$ = 0.001 111 and $\hat{\sigma}_{e_{\text{SH}}}^2$ = 0.001 360.

The estimates of the VCs above permit the solution of eq. 3, the estimated variance, and corresponding standard error. By solving eq. 3 for differing values of n and m allows generation of Table 3. As seen in Table 3, the standard errors are slightly higher for the SH regions as compared with the NPUG regions. This is because the primary variance components are the same; although the secondary variance components differ, that stage of sampling adds little to the variation. This substantiates the claim made earlier that the primary sampling units are the most vital in determining variability. It can also be seen from Table 3 that increasing n results in a much larger decrease in the standard error than does increasing m. A more detailed table could be generated by an analyst and would allow for one to pick the sample sizes required to attain a desired standard error. A more informative example of how the estimated standard error is affected by differing n and m is presented in Figs. 2 and 3. Figure 2 plots the estimated standard error versus n for dif-

Fig. 2. Plot of estimated standard error versus *n* (number of stands) for differing values of *m* (number of trees): dotted line, *m* = 5; solid line, *m* = 15; broken line, *m* = 30. (*a*) South Atlantic and Hilly regions, (*b*) North Atlantic, Piedmont, Upper Coastal, and Gulf regions.

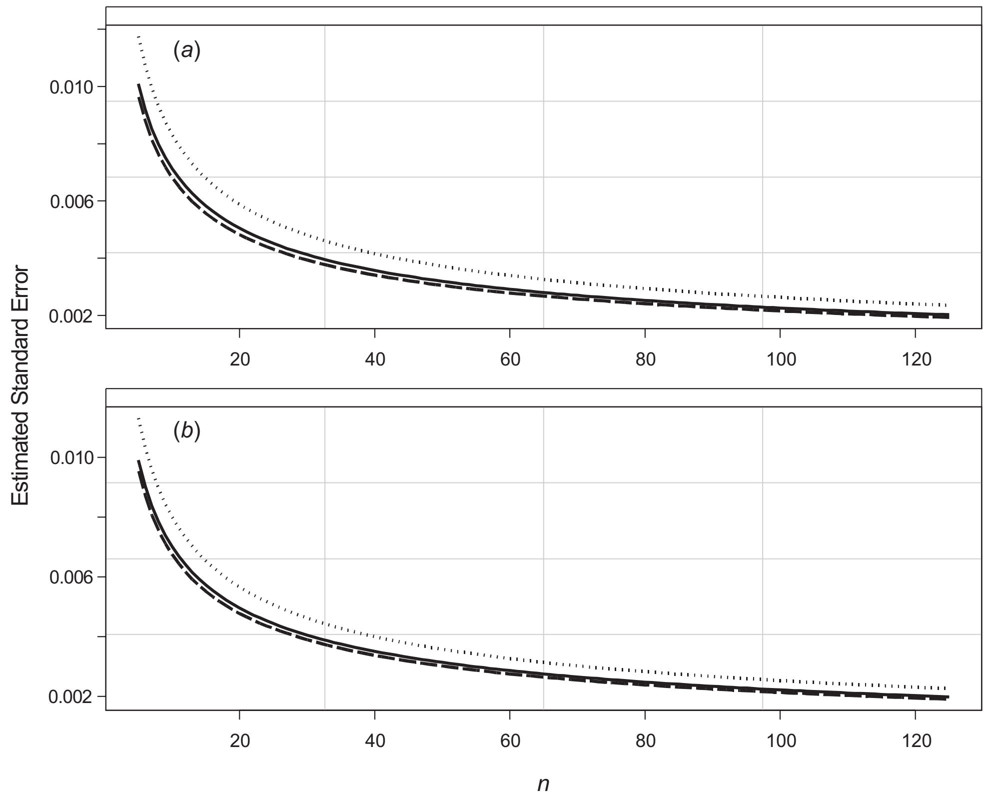
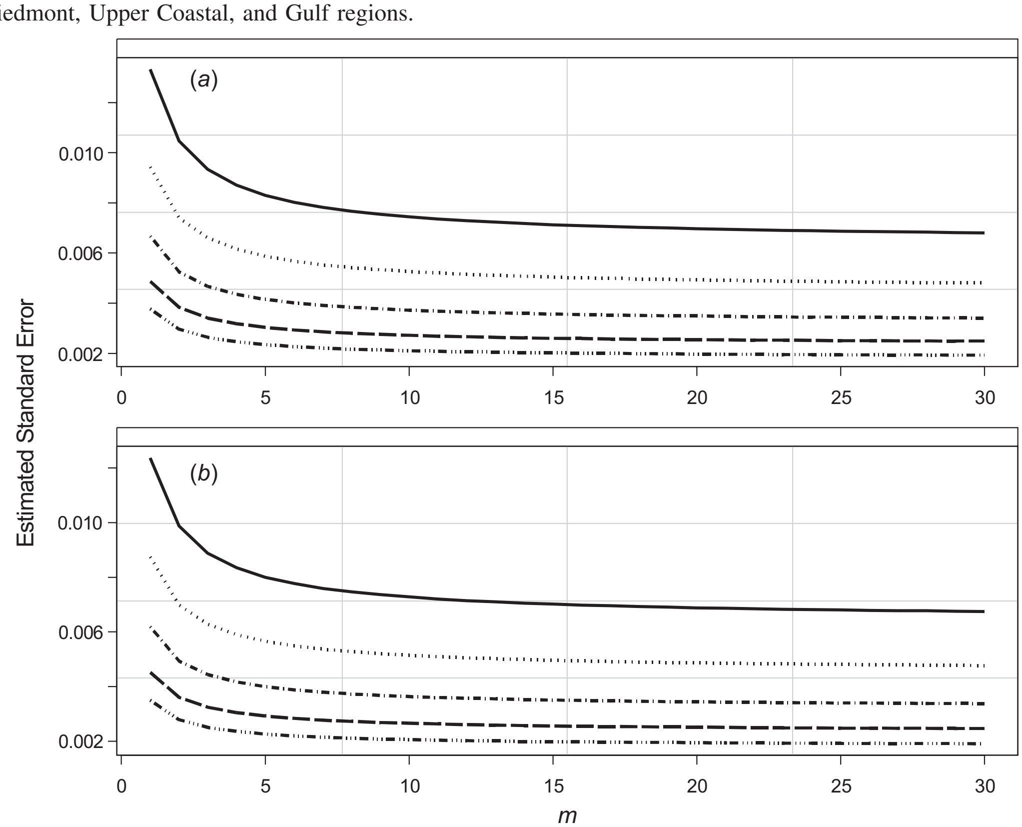


Fig. 3. Plot of estimated standard error versus *m* (number of trees) for differing values of *n* (number of stands): solid line, *n* = 10; dotted line, *n* = 20; dash–dot line, *n* = 40; broken line, *n* = 75; dash–dot–dot–dot–dash line, *n* = 125. (*a*) South Atlantic and Hilly regions, (*b*) North Atlantic, Piedmont, Upper Coastal, and Gulf regions.



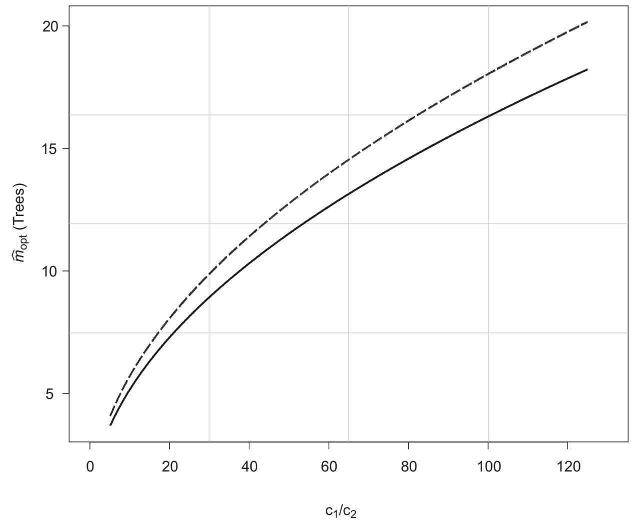
fering values of m. It can be seen that the estimated standard error decreases rapidly from 5 to 30 stands. It can also be seen in Fig. 2 that the estimated standard error decreases when selecting m = 15 or 30 compared with m = 5. This can be seen more clearly in Fig. 3, which plots the estimated standard error versus m for differing values of n. From Fig. 3, it can be seen that the estimated standard error decreases rapidly from 2 to 15 trees, after which little if any gain in the precision of the estimated standard error is gained.

As shown above, a reduction in the estimated standard error of the mean for WCSG is more influenced by the number of stands in the sample than the number of trees. However, as stated by Waters and Chester (1987), different combinations of n and m yield the same variance. For example, over a range of n = 5, 6, ..., 125 and m = 2, 3, ..., 30 and a desired standard error of 0.0045 in the SH regions, eq. 3 yields 33 combinations of n and m that achieve the desired precision, with values of nm ranging from 108 to 690 samples, thus leaving a researcher to wonder which sampling scheme is most efficient. It is now relevant to analyze the optimal allocation of sampling effort based on either precision or monetary constraints. Figure 4 utilizes eq. 7 or 8 and generates values of $\hat{m}_{\text{opt}}$ for varying cost ratios (c_1/c_2). Even when the value of c_1/c_2 is large (e.g., 125), only 18 and 21 trees are required for the NPUG and SH regions, respectively. It can also be seen in Fig. 4 that more trees are required in the SH regions compared with the NPUG regions. This is because the within-stand variation was found to be larger in the SH regions.

We then generated Table 4, which displays the optimal selection of $\hat{m}_{\text{opt}}$, $\hat{n}_{\text{opt},v}$ (eq. 7), and $\hat{n}_{\text{opt},c}$ (eq. 8), with corresponding total costs under differing values of c_1 and c_2. We fixed the precision and total cost values to be $V_{\text{f}} = 0.0045^2$ and C_{f} = \$15 000, respectively. In Table 4, the values of $\hat{n}_{\text{opt},v}$ are the optimal values that minimize the V_{f} constraint. It can be seen that for fixed V_{f}, the smaller the c_1/c_2 ratio, the allocation of funds or time is best served by increasing the number of stands to sample. The values of $\hat{n}_{\text{opt},v}$ required to obtain a precision of 0.0045 fall between 24 and 43 stands. The value of $\hat{n}_{\text{opt},c}$ is usually a fractional value, and an integer value is required. Let n be the nearest integer below $\hat{n}_{\text{opt},c}$ and that above it be n + 1. If the relation $c_1(n + 1) + c_2(n + 1)m \leq$ \$15 000 was found to be true, we select (n + 1) stands, if false we select n stands. Given a fixed cost, C_{f}, the number of stands to be sampled ($\hat{n}_{\text{opt},c}$) range from 22 to 200. For fixed cost, a minor change in the c_1/c_2 ratio dramatically affects the optimum number of stands to sample when the cost of c_1 is small.

As an aside, we calculated the mean WCSG values for each tree from ring 1 up to rings 5, 10, and 15, respectively. This allows us the opportunity to explore the among-stand and within-stand variation as if the stands were sampled at these ages. (We estimate that it takes ~3 years for these trees to reach 1.37 m; thus the true ages of the stands with trees with ages of 5, 10, and 15 years at breast height are approximately 8, 13, and 18 years, respectively.) We then performed an ANOVA for each age class to estimate the respective VCs. For simplicity, the analysis was performed across all regions. Estimates of the among-stand VC ($\hat{\sigma}_S^2$) at ages 5, 10, and 15 years were found to be 0.000 276, 0.000 534, and 0.000 682, respectively. Similarly, estimates of the within-stand VC ($\hat{\sigma}_e^2$) at ages 5, 10, and 15 years were found to be 0.001 003, 0.001 073, and 0.001 135, respectively. The increase in the among-stand VC with increasing age is not unexpected and is quite common in a longitudinal data setting. This increase is more than likely caused by stand conditions, environmental factors, edaphic factors, or weather-related factors. The differences in the VC at differing ages indicates that the standard error of the mean increases with age. Figure 5 plots the estimated standard error versus n for differing values of m and m for differing values of n at ages 5, 10, and 15 years. Figure 5 indicates sampling at younger ages reduces the estimated standard error of the mean. The most important result seen in Fig. 5 is that, regardless of age, little if any gain in the precision of the estimated standard error is gained by sampling more than 15 trees.

Fig. 4. Plot of $\hat{m}_{\text{opt}}$ versus the cost ratio, c_1/c_2, for the South Atlantic and Hilly regions (broken line) and the North Atlantic, Piedmont, Upper Coastal, and Gulf regions (solid line).



If a researcher is interested only in the value of WCSG in one stand, then the variance of the mean is given as $\hat{\sigma}^2(\hat{\mu}) = \sigma_e^2/m$, with corresponding standard error and confidence intervals of $\sqrt{\hat{\sigma}^2(\hat{\mu})}$ and $\hat{\mu} \pm t_{\alpha/2,m-1}\sqrt{\hat{\sigma}^2(\hat{\mu})}$, respectively. For example, consider that 15 trees are sampled in one stand located in the Piedmont region. Utilizing the variance components in Table 3 leads to $\hat{\sigma}^2(\hat{\mu})$ = 0.001 111 / 15, $\sqrt{\hat{\sigma}^2(\hat{\mu})} = 0.0086$. The value of the standard error here is approximately two to four times larger than those presented in Table 4, and the corresponding 95% confidence interval of $\hat{\mu} \pm 2.145(0.0086) = \hat{\mu} \pm 0.0184$ may be too wide depending on the researcher's objective. Thus, sampling a single stand does not have the benefit of variance reduction gained from combining over stands and will lead to larger variance and corresponding standard error values.

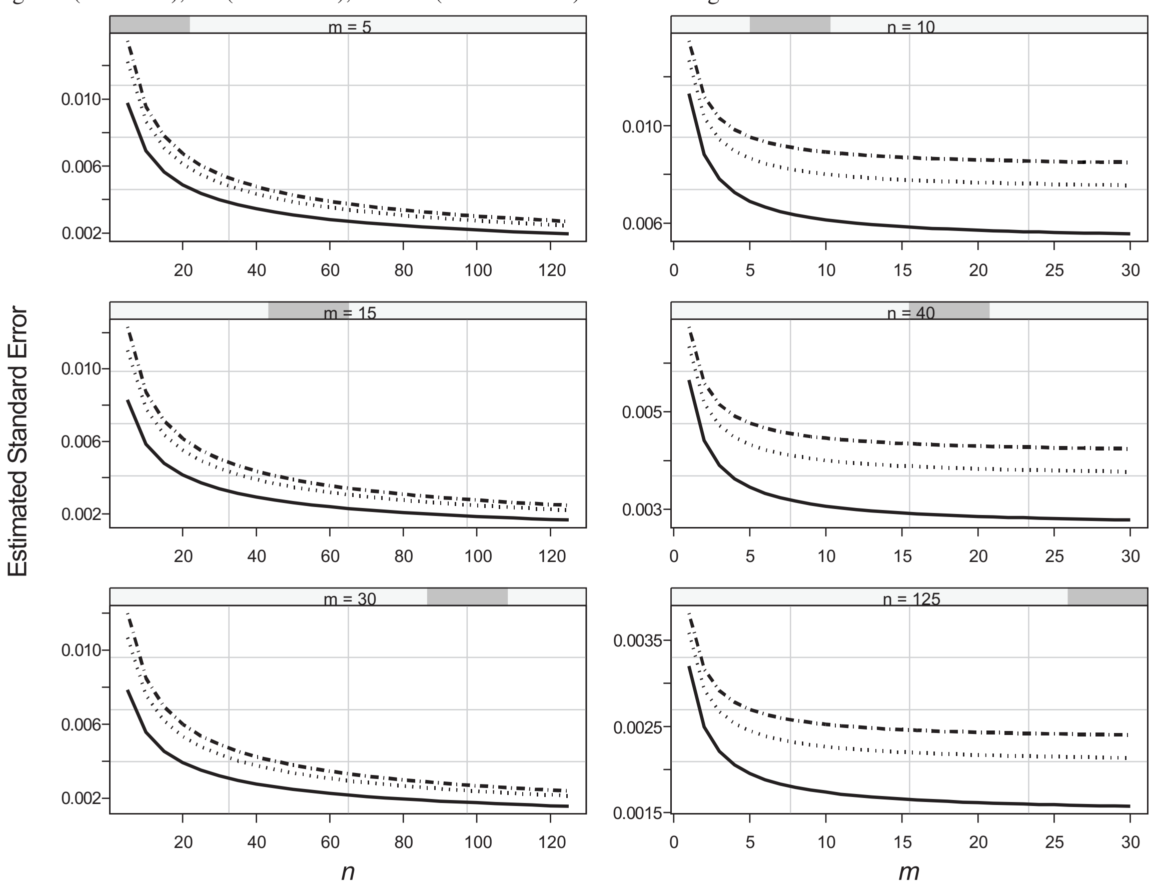
Consider a desired precision of $\sigma^2(\hat{\mu}) = 0.0045^2$, then the number of trees need to obtain this precision is calculated as $m = \sigma_e^2/\hat{\sigma}^2(\hat{\mu})$ = 0.001 111 / 0.0045^2 = 55. The standard error of the mean for a single stand will initially decrease rapidly given an increase in m then continue decreasing at a slower rate, at which point the addition of extra samples has little impact on the results. We constructed a table of

Table 4. Optimal selection of $\hat{m}_{opt}$, $\hat{n}_{opt,v}$ (eq. 7), and $\hat{n}_{opt,c}$ (eq. 8) with corresponding total costs under differing values of c_1 and c_2.

Region	c_1	c_2	c_1/c_2	$\hat{m}_{opt}$	$\hat{n}_{opt,v}$	C_v	$\hat{n}_{opt,c}$	C_c	$\sqrt{\hat{\sigma}^2(\hat{\mu}_C)}$
NPUG	50	5	10	5	32	2400	200	15 000	0.0018
		15	3.3	3	39	3 705	157	14 915	0.0022
		20	2.5	3	39	4 290	136	14 960	0.0024
	200	5	40	10	26	6 500	60	15 000	0.0030
		15	13.3	6	30	8 700	51	14 790	0.0034
		20	10	5	32	9 600	50	15 000	0.0036
	500	5	100	16	24	13 920	25	14 500	0.0044
		15	33.3	9	27	17 145	23	14 605	0.0049
		20	25	8	28	18 480	22	14 520	0.0050
SH	50	5	10	6	32	2 560	187	14 960	0.0019
		15	3.3	3	43	4 085	157	14 915	0.0024
		20	2.5	3	43	4 730	136	14 960	0.0025
	200	5	40	11	27	6 885	58	14 790	0.0031
		15	13.3	7	30	9 150	49	14 945	0.0035
		20	10	6	32	10 240	46	14 720	0.0037
	500	5	100	18	24	14 160	25	14 750	0.0044
		15	33.3	10	27	17 550	23	14 950	0.0049
		20	25	9	28	19 040	22	14 960	0.0051

Note: NPUG, North Atlantic, Piedmont, Upper Coastal, and Gulf regions; SH, South Atlantic and Hilly regions. $V_f = 0.0045^2$, C_f = \$15 000, $\sqrt{\hat{\sigma}^2(\hat{\mu}_C)}$ = estimated standard error of the mean for fixed cost (C_f), $\hat{\sigma}_S^2 = 0.000\ 418$, $\hat{\sigma}_{e_{NPUG}}^2 = 0.001\ 111$, and $\hat{\sigma}_{e_{SH}}^2 = 0.001\ 360$.

Fig. 5. Plot of estimated standard error versus *n* (stands) for differing values of *m* (trees; left column) and *m* for differing values of *n* (right column) at ages 5 (solid line), 10 (dotted line), and 15 (dash–dot line) across all regions.



$\sqrt{\hat{\sigma}^2(\hat{\mu})} = \sqrt{\sigma_e^2/m}$ for both the NPUG and SH regions, under the assumption of sampling a single stand. We then evaluated the difference in the estimated standard error at m and $m + 1$ trees to determine the point of diminishing returns. For example, in the NPUG regions, the difference in the estimated standard error from sampling 15 and 16 trees was found to be (0.0086 – 0.0083) = 0.0003, whereas the difference in the estimated standard error from sampling 55 and 56 trees was found to be (0.004 49 – 0.004 45) = 0.000 04. Although subjective, plots of these differences suggested that, for a single stand, generally no more than 45 to 55 trees should be sampled to determine WCSG.

Conclusions

The most important conclusion that can be drawn from the results of this analysis is that the number of stands sampled is more important in reducing the mean variance of WCSG than is the number of trees sampled within a stand. It is highly unlikely that any survey will attempt to sample a population as large as the one discussed here. A reduction in the size of the study area would tend to decrease the interlocation travel cost (c_1) and would lower the c_1/c_2 ratio, leading to a decrease in the optimum number of trees per stand to sample. This would permit a larger number of locations to be sampled, which would reduce the among-stand variation and result in a better estimate of the true sample mean of WCSG. It is also apparent that determination of an optimum sampling size is inherently dependent on the cost ratio and is sensitive to differing inputs. These results also suggest that the estimated standard error is reduced by sampling earlier in the life of the stands. When sampling across multiple stands (at any age), little if any gain in the precision of the standard error of the mean occurs by sampling more than 15 trees. In the general case where one is interested only in the value of WCSG in one stand, it would suffice to sample between 45 and 55 trees.

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