MATH 8080: Lie Algebras, Fall 2015

Lecture: Boyd 410, MWF 9:05–9:55 am
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Office Hours (tentative): M 1:30–2:30 pm, W 8:00–9:00 am, F 2:00–3:00 pm

Textbook: Introduction to Lie Algebras and Representation Theory, by James E. Humphreys
Additional references: Introduction to Lie Algebras, by Karin Erdmann and Mark Wildon; Lie Algebras of Finite and Affine Type, by Roger Carter; Lie Algebras, by Nathan Jacobson

Prerequisites: Mastery of linear algebra and of the methods of abstract algebra. In particular, students should have a good knowledge of notions related to eigenvalues, bilinear forms, Euclidean spaces, canonical forms, tensor products, and the basics of module theory. Much of the prerequisite theory from linear algebra can be found in Appendix A of Erdmann and Wildon’s book.

Course overview: This course in an introduction to the structure and representation theory of semisimple Lie algebras over $\mathbb{C}$. Besides appearing throughout mathematics and physics (e.g., in the study of Lie groups, differentiable manifolds, algebraic groups, quantum groups, finite groups of Lie type, particle physics), the theory of Lie algebras, and complex semisimple Lie algebras in particular, is quite beautiful. In fact, the classification theorem for complex semisimple Lie algebras served as a model for the eventual classification of all finite simple groups, one of the most important mathematical results of the twentieth century.

I anticipate that we will cover parts of Chapters I–III and V–VI of the textbook. In particular:

- Definitions and basic concepts related to Lie algebras
- Nilpotent and solvable Lie algebras (theorems of Engel, Lie, and Cartan)
- Structure of semisimple Lie algebras
- Root systems (axiomatics, the Weyl group, classification of root systems, weights)
- Universal enveloping algebras (PBW theorem)
- Classification of semisimple Lie algebras over $\mathbb{C}$
- Finite-dimensional representation theory of complex semisimple Lie algebras

My goal is for us to get as far as possible in discussing the representation theory of complex semisimple Lie algebras, and hopefully to get as far as discussing Weyl’s character formula. To get this far in the time available, we may move fairly rapidly through some parts of the material. It will occasionally be up to you to fill in gaps from the lectures by reading the relevant portions of the textbook on your own.

Grading: Grades will be assigned based on homework, attendance and participation.

Homework: Homework will be assigned and collected on a regular basis (generally bi-weekly). An up-to-date list of assignments will be maintained on the class web page.

Though I’ll only ask you to turn in a few problems from each section, I encourage you to try all of the exercises in the textbook, especially if you think your dissertation research might involve topics related to Lie theory.

Changes: The course syllabus provides a general plan for the course; deviations may be necessary. Any modifications will be prominently announced on the course website.

Thanks to Chris Drupieski, from whom this syllabus was copied (with permission!) with minor changes.