Always justify your answers, even if the question does not explicitly say so! Write your own solutions, independently of anyone else.

Core Problems: Everyone must turn these problems in.

I. Sec. 1.1 # 2, 3.

II. Prove by induction (for all \( n \in \mathbb{N} \)):
   (a) \( 1 + 5 + 9 + \cdots + (4n + 1) = (n + 1)(2n + 1) \).
   (b) \( \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1} \).

III. Sec. 1.1 # 6a [The original definition is the one at the top of p. 6. Give a combinatorial proof, as suggested in the hint.]

IV. State Rolle’s Theorem from differential calculus. Using mathematical induction, prove that if \( f \) is continuous on \([a, b]\), differentiable on \((a, b)\), and \( f \) has \( n \) zeros on \([a, b]\), then \( f' \) has at least \( n - 1 \) zeros on \([a, b]\).

V. Let \( n \) and \( k \) be positive integers. After calculating several examples, guess a closed formula [meaning no “…” or “\( \sum \)’] for
   \[
   \binom{n}{0} + \binom{n+1}{1} + \cdots + \binom{n+k}{k}.
   \]
   [It may help to stare at Pascal’s triangle while computing the sums.] Then prove your formula by induction.

Advanced Problem: Due Wed. Sep. 2. Students registered for 6000 must turn these problems in. They count for extra credit for 4000 students, but anyone hoping to get an ‘A’ in 4000 should do a reasonable number of advanced problems. Advanced problems are due one week after the core problems from the same assignment, unless announced otherwise. Please hand in Advanced Problems separately from Core Problems.

VI. Sec. 1.1 # 16 [Remark: This will be one of the harder advanced problems I’ll assign all semester. Don’t be discouraged if you don’t solve it—try next week’s advanced problem!]