Section 14.9: Total Area and Area Between Curves.

Review of Graphs.
In order to compute areas between curves, it is important that you can generate a graph for the given situation. You are expected to know the graphs of the following:

- $y = mx + b$
- $y = x^2$
- $y = |x|$
- $y = e^x$
- $y = \ln(x)$
- $y = \sqrt{x}$

You should also be able to move these graphs up/down/left/right as well as reflect/translate. See the examples below.

Example #1: Give a sketch of the following functions below.

(a) $y = -(x+5)^2 - 3$ (b) $y = \ln(x-4)$ (c) $y = \sqrt{9-x} + 3$

Total Area.
Whenever a function $f(x)$ takes on negative values on an interval $[a, b]$, the definite integral of $f(x)$ over $[a, b]$ will result in an aggregate area. This means that you might have cancelations with the negative and positive regions.

To compute the **total area** bounded by the graph of a function $y = f(x)$ and the x-axis when the function takes on both positive and negative values, you must break up the interval $[a, b]$ into monotonic subintervals. The correct total area will then be the result of the *absolute values* of the areas of each subinterval.

It will be a good idea to graph your integrand so that you can determine which sections of its graph falls above or below the x-axis.

Examples: Compute the total area of the following functions in the given interval.

1. $f(x) = x^2 - 16$ over the interval $[0, 6]$.
2. $g(x) = e^x - 1$ over the interval $[-2, 2]$.

Area Between Curves.
If $u(x)$ and $v(x)$ are continuous with $u(x) \geq v(x)$ throughout $[a, b]$, then **the area between the curves** from a to b is the integral of $(u(x) - v(x))$ from a to b:

$$A = \int_a^b [u(x) - v(x)]dx$$

This value will be a positive total area. Notice that this definition is dependent on $u(x)$ being **above** $v(x)$.

The following examples will illustrate the concept.

Examples: For the pair of functions given below, determine the area between their curves.

1. $f(x) = x + 7$ and $g(x) = 9 - x^2$.
2. $f(x) = x^3 - 6x$ and $g(x) = x^2$. (See graphic below.)