STAT 8230 — Applied Nonlinear Regression
Homework 6 – Due Thursday, Dec. 1

Homework Guidelines:

- Homework is due by 4:30 on the due date specified above. You may turn it in at the beginning of class or place it in my mailbox in the Statistics Building. **No late homeworks will be accepted without permission granted prior to the due date.**
- Use only standard (8.5×11 inch) paper and use only one side of each sheet.
- Homework should show enough detail so that the reader can clearly understand the procedures of the solutions.
- Problems should appear in the order that they were assigned.

Assignment:

1. Recall the corn yield data from p.123 of the lecture notes. The data displayed on that page are from one sample of corn plants and are really a subset of a larger data set consisting of measurements taken on several samples of corn. The complete data set can be found in the file corn2.dat on the course web site. For this problem, use the combined data from samples 8,14, 22, and 52 in corn2.dat.

    a. Fit a nonlinear mixed effect model to these data of the form:

    $$\log(y_{ij}) = \theta_{1i} - \log(1 + e^{\theta_{2i} - \theta_{3i} x_{ij}}) + e_{ij},$$

    where y_{ij} is the j^{th} measurement of dry kernel weight from the i^{th} sample, and we assume $e_{11}, \ldots$ are independent normal with mean 0 and constant variance. In addition, model $\theta_{1i}, \theta_{2i}, \theta_{3i}$ using mixed effects. That is, consider modelling θ_{1i}, for example, as $\theta_{1i} = \beta_1 + b_{1i}$ where β_1 is a fixed effect and b_{1i} is a sample-specific random effect. **As appropriate,** use mixed effects for some or all of $\theta_{1i}, \theta_{2i}, \theta_{3i}$ and model $\boldsymbol{\Psi} = \text{var}(\mathbf{b}_i)$ appropriately. Support your modelling choices with appropriate statistical and/or graphical results.

    b. Summarize your fitted model from part (a) graphically. That is, produce a plot in which the data from each sample are displayed along with both the population level and sample-level fitted curves.

    c. Write down the form of your final model in the form given by (*) on p.237 of the class notes. That is, you should follow the orange tree and indomethacin examples on pp.239–240 in how to write your model. Also give all parameter estimates and their standard errors.

2. An experiment was conducted to measure the CO_2-uptake in plants exposed to various light intensities for three different light levels (L=Low, M=Medium, H=High). Eighteen plants were randomly divided into three groups of six plants each. Each light level, low, meduium and high, was assigned to one of the groups. Then each plant was exposed to light of the assigned level at increasing light intensities over time. In consecutive 15 minute intervals, each plant received light of intensities 0, 20, 80, 150, 250, 400, 650, 900, 1300, and 2000. CO_2-uptake was measured in each 15-minute exposure. A partial listing of the data* from this experiment appears below. The data themselves can be obtained from the course web page in the file, co2.dat.

Obs. No.	Light Intensity	Plant ID	Light Level	CO_2 Uptake	Measurement Occasion
1	0	1	H	-0.43	1
2	20	1	H	0.33	2
3	80	1	H	2.65	3
4	150	1	H	6.13	4
5	250	1	H	5.90	5
6	400	1	H	7.60	6
7	650	1	H	8.70	7
8	900	1	H	9.70	8
9	1300	1	H	9.20	9
10	2000	1	H	11.52	10
11	0	2	H	-1.13	1
⋮	⋮	⋮	⋮	⋮	
178	1300	18	M	3.90	9
179	2000	18	M	4.20	10

Note that one observation was lost in this data set: the data from plant 5 at measurement occasion 8, light intensity 900, are missing. Otherwise, 10 measurements were made on each plant.

The researcher who collected these data expected that CO_2-uptake would follow an asymptotically increasing relationship with light intensity at all three light levels. She was interested in two questions: 1) Does the average upper limit of CO_2-uptake depend on light level; and 2) Does the rate of increase in the CO_2-uptake curve depend upon light level. For each of these questions, she was interested in the following secondary question: If the answer is yes, is the nature of the dependence on light level a linear trend in increasing light levels (L,M,H - assuming these are equally spaced)?

Analyze these data using nonlinear mixed effect models. Specifically, consider

* Courtesy of Heather Alley, Georgia State Botanical Gardens

nonlinear mixed models of the asymptotic regression form

$$y_{ijk} = \phi_{1ij}\left[1 - \exp\{-e^{\phi_{2ij}}(x_{ijk} - \phi_{3ij})\}\right] + e_{ijj} \qquad (*)$$

where y_{ijk} is the CO_2-uptake for the jth plant in the ith light intensity condition when measured at the k level of light intensity, and where ϕ_{hij}, $h = 1, 2, 3$ are plant specific quantities that are modeled with mixed effects to allow them to differ from plant to plant as well as across all three light intensity levels.

Thoroughly describe the process that leads to a suitable model from which you can make statistical inferences (e.g., test hypotheses) to adress the researcher's scientific questions of interest. Describe and justify the final model on which you base your inferences and summarize your results and conclusions in a thorough and appropriate manner.