

```

> Hahn <- read.table(file="hahn1.dat",header=T)
> plot(Hahn$x,Hahn$y,xlab="Temperature (K)",ylab="Coefficient of Thermal Expansion")
> title(main="Hahn Data on Thermal Expansion of Copper")
> Hahnfunc <- function(x,th1,th2,th3,th4,th5,th6,th7) {
+ (th1+x*(th2+x*(th3+x*th4)))/(1+x*(th5+x*(th6+x*th7)))
+ }
> Hahndenom <- function(x,th5,th6,th7) {
+ (1+x*(th5+x*(th6+x*th7)))
+ }
> m1Hahn.nls <- nls( y~ Hahnfunc(x,th1,th2,th3,th4,th5,th6,th7),
+ data=Hahn, trace=T,
+ start=list(th1=1,th2=-.01,th3=.005,th4=-.000001,th5=-.005,th6=.0001,th7=-.0000001))
2275325 : 1e+00 -1e-02 5e-03 -1e-06 -5e-03 1e-04 -1e-07
111131.4 : 3.488698e-01 -6.810412e-02 3.259154e-03 -2.088046e-06 -4.519875e-03 1.245972e-04 -
1.260663e-07
13936.93 : 9.257448e-01 -1.125451e-01 4.029090e-03 -3.323140e-06 -3.453484e-03 1.838449e-04 -
1.883024e-07
1757.320 : 1.771660e+00 -1.811791e-01 5.432496e-03 -4.869181e-06 -3.318363e-03 2.693938e-04 -
2.742995e-07
112.1951 : 2.003318e+00 -2.014074e-01 5.849004e-03 -5.238573e-06 -4.132285e-03 3.077984e-04 -
3.016353e-07
10.72272 : 1.715874e+00 -1.772237e-01 5.313823e-03 -4.201916e-06 -4.731940e-03 2.887176e-04 -
2.524855e-07
2.400202 : 1.362667e+00 -1.469118e-01 4.631528e-03 -2.723761e-06 -5.301264e-03 2.615125e-04 -
1.831606e-07
1.562208 : 1.136908e+00 -1.276644e-01 4.197438e-03 -1.702157e-06 -5.673459e-03 2.447869e-04 -
1.359262e-07
1.532521 : 1.082578e+00 -1.230944e-01 4.094998e-03 -1.444636e-06 -5.755338e-03 2.408815e-04 -
1.239980e-07
1.532438 : 1.077854e+00 -1.227101e-01 4.086722e-03 -1.426802e-06 -5.760833e-03 2.405521e-04 -
1.231696e-07
1.532438 : 1.077642e+00 -1.226935e-01 4.086385e-03 -1.426280e-06 -5.760992e-03 2.405378e-04 -
1.231451e-07
> m1Hahn.nls$trace
NULL
> summary(m1Hahn.nls)

```

Formula: $y \sim \text{Hahnfunc}(x, th1, th2, th3, th4, th5, th6, th7)$

Parameters:

```

Estimate Std. Error t value Pr(>|t|)
th1 1.078e+00 1.707e-01 6.313 1.40e-09 ***
th2 -1.227e-01 1.200e-02 -10.224 < 2e-16 ***
th3 4.086e-03 2.251e-04 18.155 < 2e-16 ***
th4 -1.426e-06 2.758e-07 -5.172 5.06e-07 ***
th5 -5.761e-03 2.471e-04 -23.312 < 2e-16 ***
th6 2.405e-04 1.045e-05 23.019 < 2e-16 ***
th7 -1.231e-07 1.303e-08 -9.453 < 2e-16 ***

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0818 on 229 degrees of freedom

Number of iterations to convergence: 10

Achieved convergence tolerance: 3.26e-06

```
> m2Hahn.nls <- nls(y~cbind(1,x,x^2,x^3)/Hahndenom(x,th5,th6,th7),
+ data=Hahn, trace=T,alg="plinear",
+ start=list(th5=-.005,th6=.0001,th7=-.0000001))
52.37535 : -5.000000e-03 1.000000e-04 -1.000000e-07 4.246432e-01 -1.384956e-02 1.638663e-03 -
1.641111e-06
13.16452 : -2.685126e-03 1.672625e-04 -1.668996e-07 1.020464e-01 -5.126586e-02 3.075557e-03 -
2.952519e-06
4.465369 : -3.413850e-03 2.848764e-04 -2.660818e-07 1.728468e+00 -1.787342e-01 5.446743e-03 -
4.762388e-06
1.927021 : -4.799327e-03 2.848453e-04 -2.105583e-07 1.697482e+00 -1.737164e-01 5.192301e-03 -
3.353825e-06
1.537299 : -5.717978e-03 2.437646e-04 -1.312889e-07 1.135973e+00 -1.269225e-01 4.170147e-03 -
1.606482e-06
1.532444 : -5.758512e-03 2.407192e-04 -1.234895e-07 1.080395e+00 -1.229078e-01 4.090781e-03 -
1.433753e-06
1.532438 : -5.760948e-03 2.405432e-04 -1.231536e-07 1.077724e+00 -1.226997e-01 4.086509e-03 -
1.426459e-06
1.532438 : -5.760993e-03 2.405375e-04 -1.231447e-07 1.077637e+00 -1.226931e-01 4.086378e-03 -
1.426271e-06
> m2Hahn.nls$trace
NULL
> coef(m2Hahn.nls)
      th5      th6      th7      .lin1      .lin.x
-5.760993e-03 2.405375e-04 -1.231447e-07 1.077637e+00 -1.226931e-01
      .lin3      .lin4
4.086378e-03 -1.426271e-06
> th5hat<-coef(m2Hahn.nls)[1]
> th6hat<-coef(m2Hahn.nls)[2]
> th7hat<-coef(m2Hahn.nls)[3]
> Ahat<-cbind(1,Hahn$x,Hahn$x^2,Hahn$x^3)/Hahndenom(Hahn$x,th5hat,th6hat,th7hat)
> lm(Hahn$y~-1+Ahat[,1]+Ahat[,2]+Ahat[,3]+Ahat[,4])
```

Call:

```
lm(formula = Hahn$y ~ -1 + Ahat[, 1] + Ahat[, 2] + Ahat[, 3] + Ahat[, 4])
```

Coefficients:

```
Ahat[, 1] Ahat[, 2] Ahat[, 3] Ahat[, 4]
1.078e+00 -1.227e-01 4.086e-03 -1.426e-06
```

```
> m3Hahn.nls <- nls(y~cbind(1,x,x^2,x^3)/Hahndenom(x,th5,th6,th7),
```

```

+ data=Hahn, trace=T,alg="plinear",
+ start=list(th5=0,th6=0,th7=0))
242.8337 : 0.000000e+00 0.000000e+00 0.000000e+00 -6.846356e-01 1.168826e-01 -2.317749e-04
1.483604e-07
59.16408 : 4.886705e-03 -5.178239e-06 1.611463e-09 -4.405437e+00 2.150243e-01 -3.347754e-04
1.879325e-07
33.12517 : 6.123432e-03 1.731088e-05 -2.182375e-08 -5.117029e+00 2.168630e-01 1.300384e-04 -
2.602496e-07
10.83850 : 1.262406e-03 1.581001e-04 -1.588905e-07 -1.362769e+00 2.593646e-03 2.989783e-03 -
2.870776e-06
3.451725 : -2.937073e-03 3.266475e-04 -2.986007e-07 2.135869e+00 -2.177791e-01 6.312062e-03 -
5.365807e-06
1.641796 : -5.431570e-03 2.621012e-04 -1.674893e-07 1.417459e+00 -1.488240e-01 4.621940e-03 -
2.398903e-06
1.532999 : -5.739851e-03 2.418893e-04 -1.262177e-07 1.099692e+00 -1.243685e-01 4.120650e-03 -
1.493928e-06
1.532439 : -5.760400e-03 2.405930e-04 -1.232397e-07 1.078478e+00 -1.227581e-01 4.087687e-03 -
1.428308e-06
1.532438 : -5.760983e-03 2.405390e-04 -1.231470e-07 1.077660e+00 -1.226949e-01 4.086413e-03 -
1.426319e-06
1.532438 : -5.760994e-03 2.405374e-04 -1.231445e-07 1.077636e+00 -1.226930e-01 4.086376e-03 -
1.426267e-06
> m3Hahn.nls$trace
NULL
> coef(m3Hahn.nls)
      th5      th6      th7      .lin1      .lin.x
-5.760994e-03 2.405374e-04 -1.231445e-07 1.077636e+00 -1.226930e-01
      .lin3      .lin4
4.086376e-03 -1.426267e-06
> x0<-seq(from=min(Hahn$x),to=max(Hahn$x),by=.5)
> y0<-Hahnfunc(x0,coef(m1Hahn.nls)[1],coef(m1Hahn.nls)[2],coef(m1Hahn.nls)[3],
+ coef(m1Hahn.nls)[4],th5hat,th6hat,th7hat)
> lines(x0,y0)
>

```

Hahn Data on Thermal Expansion of Copper

