

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \Rightarrow (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Figure 2.14 Verifying that $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology

Example 2.19 For statements P and Q , determine whether the compound statement

$$(P \wedge (\sim Q)) \Leftrightarrow (P \Rightarrow Q)$$

is a tautology, a contradiction or neither.

Solution The truth table in Figure 2.15 shows that the compound statement $(P \wedge (\sim Q)) \Leftrightarrow (P \Rightarrow Q)$ is a contradiction. \blacklozenge

P	Q	$\sim Q$	$P \wedge (\sim Q)$	$P \Rightarrow Q$	$(P \wedge (\sim Q)) \Leftrightarrow (P \Rightarrow Q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

Figure 2.15 Verifying that $(P \wedge (\sim Q)) \Leftrightarrow (P \Rightarrow Q)$ is a contradiction

SECTION 2.7 EXERCISES

- 2.46. For statements P and Q , show that $P \Rightarrow (P \vee Q)$ is a tautology.
- 2.47. For statements P and Q , show that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ and $(P \Rightarrow \sim Q) \wedge (P \wedge Q)$ are contradictions.
- 2.48. For statements P and Q , show that $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is a tautology. Then state $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ in words. (This is an important logical argument form, called **modus ponens**.)
- 2.49. For statements P , Q and R , show that $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a tautology. Then state this compound statement in words. (This is another important logical argument form, called **sylogism**.)
- 2.50. Let R and S be compound statements involving the same component statements. If R is a tautology and S is a contradiction, then what can be said of the following?
- (a) $R \vee S$ (b) $R \wedge S$ (c) $R \Rightarrow S$ (d) $S \Rightarrow R$.
- 2.51. For statements P and Q , determine whether the compound statement $(P \vee Q) \vee (Q \Rightarrow P)$ is a tautology, a contradiction or neither.
- 2.52. For statements P and Q , determine whether the compound statement $((P \Rightarrow Q) \Rightarrow P) \Rightarrow (P \Rightarrow (Q \Rightarrow P))$ is a tautology, a contradiction or neither.