1.46. Determine the following:

(a) $\bigcup_{n=1}^{\infty}\left(-\frac{1}{n}, \frac{1}{n}\right)$ and $\bigcap_{n=1}^{\infty}\left(-\frac{1}{n}, \frac{1}{n}\right)$

(b) $\bigcup_{n=1}^{\infty}\left[\frac{n-1}{n}, \frac{n+1}{n}\right]$ and $\bigcap_{n=1}^{\infty}\left[\frac{n-1}{n}, \frac{n+1}{n}\right]$

1.47. Determine the following:

(a) $\bigcup_{n=1}^{\infty}\left\{\sin^2\frac{n\pi}{2} + \cos^2\frac{n\pi}{2}\right\}$ and $\bigcap_{n=1}^{\infty}\left\{\sin^2\frac{n\pi}{2} + \cos^2\frac{n\pi}{2}\right\}$

(b) $\bigcup_{n=1}^{\infty}\left\{\sin\frac{n\pi}{2} + \cos\frac{n\pi}{2}\right\}$ and $\bigcap_{n=1}^{\infty}\left\{\sin\frac{n\pi}{2} + \cos\frac{n\pi}{2}\right\}$

1.5 PARTITIONS OF SETS

Recall that two sets are disjoint if their intersection is the empty set. A collection $\mathcal{S}$ of subsets of a set A is called **pairwise disjoint** if every two distinct subsets that belong to $\mathcal{S}$ are disjoint. For example, let $A = \{1, 2, \ldots, 7\}$, $B = \{1, 6\}$, $C = \{2, 5\}$, $D = \{4, 7\}$ and $S = \{B, C, D\}$. Then S is a pairwise disjoint collection of subsets of A since $B \cap C = B \cap D = C \cap D = \emptyset$. On the other hand, let $A' = \{1, 2, 3\}$, $B' = \{1, 2\}$, $C' = \{1, 3\}$, $D' = \{2, 3\}$ and $S' = \{B', C', D'\}$. Although S' is a collection of subsets of A' and $B' \cap C' \cap D' = \emptyset$, the set S' is *not* a pairwise disjoint collection of sets since $B' \cap C' \neq \emptyset$, for example. Indeed, $B' \cap D'$ and $C' \cap D'$ are also nonempty.

We will often have the occasion (especially in Chapter 9) to encounter, for a nonempty set A, a collection $\mathcal{S}$ of pairwise disjoint nonempty subsets of A with the added property that every element of A belongs to some subset in $\mathcal{S}$. Such a collection is called a **partition** of A. A **partition** of A can also be defined as a collection $\mathcal{S}$ of nonempty subsets of A such that every element of A belongs to exactly one subset in $\mathcal{S}$. Furthermore, a partition of A can be defined as a collection $\mathcal{S}$ of subsets of A satisfying the three properties:

(1) $X \neq \emptyset$ for every set $X \in \mathcal{S}$;

(2) for every two sets $X, Y \in \mathcal{S}$, either $X = Y$ or $X \cap Y = \emptyset$;

(3) $\bigcup_{X \in \mathcal{S}} X = A$.

Example 1.22 Consider the following collections of subsets of the set $A = \{1, 2, 3, 4, 5, 6\}$:

$$S_1 = \{\{1, 3, 6\}, \{2, 4\}, \{5\}\};$$
$$S_2 = \{\{1, 2, 3\}, \{4\}, \emptyset, \{5, 6\}\};$$
$$S_3 = \{\{1, 2\}, \{3, 4, 5\}, \{5, 6\}\};$$
$$S_4 = \{\{1, 4\}, \{3, 5\}, \{2\}\}.$$

Determine which of these sets are partitions of A.

Solution The set S_1 is a partition of A. The set S_2 is not a partition of A since $\emptyset$ is one of the elements of S_2. The set S_3 is not a partition of A either since the element 5 belongs to

1.69. Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Determine the following:

(a) $(A \times B) \cap (B \times A)$ (b) $(A \cap B) \times (B \cap A)$

1.70. Let $R = \{(1, 1), (2, 1), (3, 2), (3, 3), (4, 2), (4, 3)\}$ be a collection of ordered pairs. Find subsets A, B, C, D of the set $\{1, 2, 3, 4\}$ such that $R = ((A \times B) \cup (C \times D)) - (D \times D)$.

1.71. For $i = 1, 2, 3$, let $A_i = \{i, i + 1\}$. Determine $\left|\cup_{i=1}^{3} (A_i \times A_i)\right|$

1.72. For $A = \{1, 2\}$ and $B = \{3, 4\}$, it follows that $P = \{A, B\}$ is a partition of the set $S = \{1, 2, 3, 4\}$. Describe a partition of $S \times S$ in terms of Cartesian products of the sets A and B.

Chapter 1 Supplemental Exercises

The Chapter Presentation for Chapter 1 can be found at **goo.gl/jxCZz4**

1.73. The set $T = \{2k + 1 : k \in \mathbf{Z}\}$ can be described as $T = \{\ldots, -3, -1, 1, 3, \ldots\}$. Describe the following sets in a similar manner.

(a) $A = \{4k + 3 : k \in \mathbf{Z}\}$ (b) $B = \{5k - 1 : k \in \mathbf{Z}\}$.

1.74. Let $S = \{-10, -9, \ldots, 9, 10\}$. Describe each of the following sets as $\{x \in S : p(x)\}$, where $p(x)$ is some condition on x.

(a) $A = \{-10, -9, \ldots, -1, 1, \ldots, 9, 10\}$ (b) $B = \{-10, -9, \ldots, -1, 0\}$
(c) $C = \{-5, -4, \ldots, 0, 1, \ldots, 7\}$ (d) $D = \{-10, -9, \ldots, 4, 6, 7, \ldots, 10\}$.

1.75. Describe each of the following sets by listing its elements within braces.

(a) $\{x \in \mathbf{Z} : x^3 - 4x = 0\}$ (b) $\{x \in \mathbf{R} : |x| = -1\}$
(c) $\{m \in \mathbf{N} : 2 < m \leq 5\}$ (d) $\{n \in \mathbf{N} : 0 \leq n \leq 3\}$
(e) $\{k \in \mathbf{Q} : k^2 - 4 = 0\}$ (f) $\{k \in \mathbf{Z} : 9k^2 - 3 = 0\}$
(g) $\{k \in \mathbf{Z} : 1 \leq k^2 \leq 10\}$.

1.76. Determine the cardinality of each of the following sets.

(a) $A = \{1, 2, 3, \{1, 2, 3\}, 4, \{4\}\}$ (b) $B = \{x \in \mathbf{R} : |x| = -1\}$
(c) $C = \{m \in \mathbf{N} : 2 < m \leq 5\}$ (d) $D = \{n \in \mathbf{N} : n < 0\}$
(e) $E = \{k \in \mathbf{N} : 1 \leq k^2 \leq 100\}$ (f) $F = \{k \in \mathbf{Z} : 1 \leq k^2 \leq 100\}$.

1.77. For $A = \{-1, 0, 1\}$ and $B = \{x, y\}$, determine $A \times B$.

1.78. Let $U = \{1, 2, 3\}$ be the universal set and let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{1, 3\}$. Determine the following.

(a) $(A \cup B) - (B \cap C)$ (b) $\overline{A}$ (c) $\overline{B \cup C}$ (d) $A \times B$.

1.79. Let $A = \{1, 2, \ldots, 10\}$. Give an example of two sets S and B such that $S \subseteq \mathcal{P}(A)$, $|S| = 4$, $B \in S$ and $|B| = 2$.

1.80. For $A = \{1\}$ and $C = \{1, 2\}$, give an example of a set B such that $\mathcal{P}(A) \subset B \subset \mathcal{P}(C)$.

1.81. Give examples of two sets A and B such that $A \cap \mathcal{P}(A) \in B$ and $\mathcal{P}(A) \subseteq A \cup B$.

1.82. Which of the following sets are equal?

$A = \{n \in \mathbf{Z} : -4 \leq n \leq 4\}$ $D = \{x \in \mathbf{Z} : x^3 = 4x\}$
$B = \{x \in \mathbf{N} : 2x + 2 = 0\}$ $E = \{-2, 0, 2\}$.
$C = \{x \in \mathbf{Z} : 3x - 2 = 0\}$